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A family of macroscopic node models

Erik-Sander Smits [a,*], Michiel C.J. Bliemer [b], Adam J. Pel [a], Bart van Arem [a]

[a] Department of Transport & Planning, Delft University of Technology, Postbus 5048, 2600 GA Delft, The Netherlands
[b] Institute of Transport and Logistics Studies, The University of Sydney, NSW 2006, Australia

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ABSTRACT

The family of macroscopic node models which comply to a set of basic requirements is presented and analysed. Such models are required in macro-, mesoscopic traffic flow models, including dynamic network loading models for dynamic traffic assignment. Based on the behaviour of drivers approaching and passing through intersections, the model family is presented. The headway and the turn delay of vehicles are key variables. Having demand and supply as input creates a natural connection to macroscopic link models. Properties like the invariance principle and the conservation of turning fractions are satisfied. The inherent non-uniqueness is analysed by providing the complete set of feasible solutions. The node models proposed by Tampère et al. (2011), Flötteröd and Rohde (2011) and Gibb (2011) are members of the family. Furthermore, two new models are added to the family. Solution methods for all family members are presented, as well as a qualitative and quantitative comparison. Finally, an outlook for the future development of empirically verified models is given.

1. Introduction and background to macroscopic node models

A core component of every dynamic transportation model is to compute the time-varying traffic conditions (described by, e.g., flows, densities, headways, speeds, travel times, etc.) on a network once the dynamic travel demand from origins to destinations is given. This traffic simulation procedure is often referred to as dynamic network loading (DNL). Hence, the main purpose of DNL models is to determine the emerging traffic conditions as a result of the interaction between infrastructure supply and travel demand. To this end, DNL models typically consist of a link model and a node model.

The link model computes the dynamic traffic flow propagation along homogeneous road stretches, while the node model computes the traffic conditions at discontinuities in the network, such as bottlenecks and intersections. For the link model, fundamental diagrams are used to describe the traffic dynamics and underlying driving behaviour. Depending on the level of aggregation in the representation of traffic, models can be categorised as microscopic, mesoscopic, or macroscopic, and similarly the fundamental diagrams describe relationships for pace-headway, spacing-speed, and density-flow (Laval and Leclercq, 2013).

Unfortunately, for nodes no general driving behaviour representative (as the fundamental diagram is for links) is known. Nevertheless, this study shows that cumulative flow curves on the incoming links, turns, and outgoing links of a node can be derived and provide a complete representation of the traffic dynamics. Furthermore, this paper shows how these cumulative flow curves can be used to describe driving behaviour at nodes in terms of time-headways. The benefit hereof is twofold.

* Corresponding author. Tel.: +31 (0)15 2784977.
E-mail address: e.smits@tudelft.nl (E.-S. Smits).

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First, this newly introduced way to represent traffic at nodes allows to derive and analyse the full family of node models consistent with the requirements for the *Generic Class of first-order Node Models* (GCNM) as presented by Tampère et al. (2011). Second, this representation of traffic based on a time-headway relationship yields descriptive variables that can be interpreted at the level of individual driving behaviour. Hence, earlier developed models belonging to the GCNM can now be analysed according to their assumptions on the underlying driving behaviour at nodes. Furthermore, new models fulfilling the requirements for the GCNM can be derived based on explicit behavioural assumptions.

In the transition from static to dynamic transportation models, the field of traffic flow theory has studied the propagation of traffic dynamics along homogeneous road stretches (links) exhaustively. However, where the link model propagates these traffic conditions along the links, the node model determines most congestion seeds where queues originate (due to insufficient downstream capacity) as well as the direction (i.e. the upstream links) towards which these queues spill back. The validity of the node model is therefore particularly important for traffic assignment studies and traffic flow on dense (urban) networks. Although node models have been studied in several papers through the last decades, they received significantly less attention than link models. Early contributions on node models include (Daganzo, 1995; Lebacque, 1996), and were in the next decade followed by (Jin and Zhang, 2003, 2004; Ni and Leonard, 2005; Bliemer, 2007; Jin, 2010, 2012a).

Several studies have posed requirements for the validity of a node model. Lebacque and Khoshyaran (2005) identified two invariance principles to ensure consistency between traffic flows on links and nodes. The *first invariance principle* requires that when spillback occurs on an upstream link, the outflow of that link is invariant to an increase of the upstream demand. The *second invariance principle* requires that when the supply (or capacity) is not fully utilized at a downstream link, the inflow of that link is invariant to a decrease in the supply. Later, Tampère et al. (2011) constructed a complete set of requirements for node models including these invariance principles as well as demand and supply constraints, requirements for the conservation of turning fractions (due to first-in first-out), and individual flow maximization. Also general applicability, i.e. any number of in- and outlinks, is a requirement. If a node model satisfies these requirements it belongs to the earlier mentioned *Generic Class of first-order Node Models* (GCNM); call this set the *generic requirements*.

Tampère et al. (2011) argue that if any of the generic requirements is violated, the model is either not applicable on general networks, or does not comply with basic traffic characteristics. Therefore, the generic requirements are necessary to ensure the behavioural validity of a generally applicable node model with any number of in- and outlinks. At present only two node models exist that satisfy the generic requirements, namely the model presented by Tampère et al. (2011) and Flötteröd and Rohde (2011), and another model by Gibb (2011). The generic requirements are not sufficient to guarantee a unique solution. In order to create a specific node model, additional constraints need to be introduced. Those determine how the downstream supply is distributed. In the capacity proportional model of Tampère et al. (2011) and Flötteröd and Rohde (2011) the capacity of outgoing links is divided among competing flows proportional to the capacity[1] of the corresponding incoming links. This is achieved in (Tampère et al., 2011) by adding a capacity proportional supply constraint interaction rule (SCIR) to the total flow maximization problem. In (Flötteröd and Rohde, 2011) instead, the incremental transfer principle is used where the incremental node model solution is the stationary point of a dynamic system. Notwithstanding that these two models are derived from different principles, their solutions are identical. In the capacity consumption equivalence model by Gibb (2011) the underlying assumption is that traffic towards a saturated (downstream) link consumes more capacity of its (upstream) link. Although both node models satisfy all requirements, they lack behavioural foundation (the first model to a larger extent than the latter), complicating a assessment of their validity. Furthermore, non-iterative and non-repetitive[2] solution methods are lacking for both models which, especially in Gibb's case, lead to long calculation times. Finally, the formulations of these models are not compatible as they build on different (algorithm driven) variables, making it impossible to capture one model in the others' framework.

This paper (i) presents the representation of traffic flows at nodes according to the time-headway relationship and turn delays (a concept that will be introduced in more detail in this paper) as decision variables (ii) shows how all models belonging to the GCNM (i.e., satisfying the generic requirements) fit into this framework yielding a family of node models where the challenge for each model is to find a set of turn delays, (iii) presents two new node models, (iv) shows how the two existing node models by Tampère et al. (2011), Flötteröd and Rohde (2011) and Gibb (2011) are specific cases of the family, (v) presents the mathematical optimization problem for the node model family as multi-objective optimization problem and discusses Pareto optimality and the output relevant feasible solution set,[3] (vi) provides solution methods for all models, and (vii) analyses the family of node models (including specific cases) on an illustrative three-leg node.

In the past other models are presented that do no satisfy all generic requirements. Corthout (2012, Chapter 3, Tables 1–3) provides an extensive literature overview of all models that do not satisfy all requirements; Tables 1–3 shows which models satisfy which requirements. It should be noted that the problem was already solved for merges and diverges[4] by Daganzo (1995). These nodes have respectively one outgoing or one incoming link. For the merge a priority parameter is required. Node models that do not satisfy the invariance principles may lead to non-stationary turn flow rates in kinematic wave models. For example, calculating an exact solution to the first-order kinematic wave model in continuous time with a node model that does not satisfy the first invariance principle, and using an event-based algorithm such as presented in Raadsen et al. (2014) leads to

[1] More precisely, the *directed* capacity, as explained in depth in Section 4.3.
[2] I.e., does not repeat similar calculations.
[3] Section 5.1 clarifies what output relevant solutions are.
[4] These are the most important nodes at highways, and widely used in traffic flow theory.