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Boundedly rational user equilibria (BRUE): Mathematical formulation and solution sets ☆

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ABSTRACT

Boundedly rational user equilibria (BRUE) represent traffic flow distribution patterns where travellers can take any route whose travel cost is within an 'indifference band' of the shortest path cost. Those traffic flow patterns satisfying the above condition constitute a set, named the BRUE solution set. It is important to obtain all the BRUE flow patterns, because it can help predict the variation of the link flow pattern in a traffic network under the boundedly rational behavior assumption. However, the methodology of constructing the BRUE set has been lacking in the established literature. This paper fills the gap by constructing the BRUE solution set on traffic networks with fixed demands. After defining ε-BRUE, where ε is the indifference band for the perceived travel cost, we formulate the ε-BRUE problem as a nonlinear complementarity problem (NCP), so that a BRUE solution can be obtained by solving a BRUE–NCP formulation. To obtain the BRUE solution set encompassing all BRUE flow patterns, we propose a methodology of generating acceptable path set which may be utilized under the boundedly rational behavior assumption. We show that with the increase of the indifference band, the acceptable path set that contains boundedly rational equilibrium flows will be augmented, and the critical values of indifference band to augment these path sets can be identified by solving a family of mathematical programs with equilibrium constraints (MPEC) sequentially. The BRUE solution set can then be obtained by assigning all traffic demands to the acceptable path set. Various numerical examples are given to illustrate our findings.

1. Introduction

In the static traffic assignment problem, the traditional 'perfect rationality'(PR) route choice paradigm (Wardrop, 1952) assumes that given the travel cost of each road, travellers only take the shortest paths (i.e., the least utility paths). However, this assumption is very restrictive in reality. Many studies, including simulation experiments (Nakayama et al., 2001), stated preference surveys (Avineri and Prashker, 2004), and revealed preference surveys through GPS tracking (Morikawa et al., 2005; Zhu, 2011) showed that in reality drivers do not always choose the shortest paths, and the classical Wardrop user equilibrium (UE) assignment model cannot give accurate prediction of traffic flow patterns. Thus, many other behavioral models have been developed to relax the PR assumption.

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As opposed to 'rationality as optimization,' Herbert Simon, in 1957, proposed that people are boundedly rational in their decision-making process. This is either because people lack accurate information, or they are incapable of obtaining the optimized decision due to the complexity of the situations. They tend to seek a satisfactory choice solution instead. Since then, the concept of 'bounded rationality' (BR) was studied extensively in Economics and Psychology literature (see Conlisk, 1996 for a detailed review).

Mahmassani and Chang (1987) introduced the BR assumption for the first time in modeling pre-trip departure time selection for a single bottleneck. Since then, a large body of literature incorporated bounded rationality in various transportation models, such as route choice behavior (Han and Timmermans, 2006; Nakayama et al., 2001), traffic safety (Sivak, 2002), hyperpath assignment (Fonzone and Bell, 2010), transportation planning (Gifford and Checherita, 2007; Khisty and Arslan, 2005), traffic policy making (Marsden et al., 2012) and so on. Some researchers also incorporated some thresholds to the discrete choice model to capture the impact of inertia on travellers' choices (Cantillo et al., 2006, 2007). All these studies indicated that the BR assumption plays a very important role in transportation modeling.

When the BR assumption is used to model drivers' route choice behavior, there are two aspects regarding the boundedly rational route choice process. Some studies suggested that travellers do not take the shortest paths because they are not capable of perceiving actual travel costs due to limited cognitive capacity, or it is too costly to search information about all alternative paths (Gabaix et al., 2006; Gao et al., 2011).

On the other hand, some studies assumed that all path cost information are available to travellers through some information system, but they will not switch to shorter paths due to the existence of inertia, which was quantified by a term named 'indifference band' (Mahmassani and Chang, 1987). A series of experiments were conducted by Mahmassani and his colleges to validate this BR behavioral assumption and calibrate the values of indifference bands (Hu and Mahmassani, 1997; Jayakrishnan et al., 1994a; Mahmassani and Chang, 1987; Mahmassani and Jayakrishnan, 1991; Mahmassani and Liu, 1999; Srinivasan and Mahmassani, 1999). These experiments were conducted on an interactive simulator-DYNASMART, incorporating pre-trip departure time, route choices and en-route path switching decisions. Subjects, as travellers, could change paths en-route at each node and also adjust their departure-time choices the next day based on the previous days' travel experiences. Travellers were assumed to follow the BR behavioral rule in decision-making processes, i.e., they would only switch routes when the improved trip time exceeded some indifference bands. The values of these indifference bands depended on individual characteristics and network performances. Lu and Mahmassani (2008) further studied the impact of the congestion pricing on drivers' behavior within the boundedly rational behavioral framework.

In this study, we assume that travellers can perceive travel costs accurately but some indifference bands exist due to inertia to switch routes. When traffic flow patterns stabilize to some equilibrium, called 'boundedly rational user equilibria' (BRUE), travellers can take any route whose travel time is within an indifference band of the shortest path cost (Guo and Liu, 2011; Lou et al., 2010). Indifference bands vary among origin–destination (OD) pairs. By introducing one parameter (indifference band) for each OD pair, the BR framework relaxes the restrictive PR assumption that travellers only take the shortest paths at equilibrium.

According to Ben-Akiva et al. (1984), travellers' route choice behavior is regarded as a two-stage process: path set generation (i.e., a path choice set is generated between origin and destination according to route characteristics) and traffic assignment (i.e., the traffic demands are mapped to these generated paths based on certain traffic assignment criteria). Accordingly, in this paper, we will study how to generate boundedly rational path sets first, and then assign traffic demands to these paths based on the BRUE condition. The BRUE solution set is constructed by using networks with fixed demand. Obtaining the BRUE solution set and exploring the fundamental mathematical properties of BRUE will serve as a building block for BRUE related applications, such as BR-related congestion pricing and other network design problems.

Following the two-stage route choice process, the rest of the paper is organized as follows: In Section 2, the ε-BRUE is defined and formulated as a nonlinear complementarity problem (NCP). In Section 3, the BRUE-related acceptable path set is defined, and its structure is studied. In Section 4, how to obtain the acceptable path set is presented. In Section 5, we will construct the BRUE path flow solution set based on the acceptable path set. Some examples are given to illustrate the structure of the BRUE path flow solution set. Conclusions and future work are discussed in Section 6.

2. Definition of ε-BRUE and nonlinear complementarity formulation

The traffic network is represented by a directed graph that includes a set of consecutively numbered nodes, $\mathcal{N}$, and a set of consecutively numbered links, $\mathcal{L}$. Let $\mathcal{W}$ denote the O-D pair set connected by a set of simple paths (composed of a sequence of distinct nodes), $\mathcal{P}^w$, through the network. The traffic demand for OD pair w is d^w. Let f_i^w denote the flow on path $i \in \mathcal{P}$ for OD pair w, then the path flow vector is $\mathbf{f} = \{f_i^w\}_{i\in\mathcal{P}^w}^{w\in\mathcal{W}}$. The feasible path flow set is to assign the traffic demand on the feasible paths: $\mathcal{F} \triangleq \{\mathbf{f} : \mathbf{f} \geqslant \mathbf{0}, \sum_{i\in\mathcal{P}} f_i^w = d^w, \forall w \in \mathcal{W}\}$. Denote x_a as the link flow on link a, then the link flow vector is $\mathbf{x} = \{x_a\}_{a\in\mathcal{L}}$. Each link $a \in \mathcal{L}$ is assigned a cost function of the link flow, written as $t(\mathbf{x})$. Let $\delta_{a,i}^w = 1$ if link a is on path i connecting OD pair w, and 0 if not; then $\Delta \triangleq \{\delta_{a,i}^w\}_{a\in\mathcal{L},i\in\mathcal{P}}^{w\in\mathcal{W}}$, denotes the link-path incidence matrix. Therefore $f_i^w = \sum_a \delta_{a,i}^w x_a$, and it can be rewritten in a vector form as $\mathbf{x} = \Delta\mathbf{f}$. Denote $C_i^w(\mathbf{f})$ as the path cost on path i for OD pair w, then the path cost vector $C(\mathbf{f}) \triangleq \{C_i^w(\mathbf{f})\}_{i\in\mathcal{P}}^{w\in\mathcal{W}}$. So $C(\mathbf{f}) = \Delta^T t(\mathbf{x})$ under the additive path cost assumption.

In this paper, we assume the link cost is separable, continuous and linear with respect to its own link flow, i.e., $t(\mathbf{x}) = H\mathbf{x}$, where H is the Jacobian matrix of the link cost. Then the path cost can be computed as: $C(\mathbf{f}) = \Delta^T t(\mathbf{x}) = \Delta^T H \Delta\, \mathbf{f} \triangleq A\mathbf{f}$.