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Transit assignment: Approach-based formulation, extragradient method, and paradox

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A B S T R A C T

This paper uses the concept of approach proportion to propose a novel variational inequality (VI) formulation of the frequency-based transit assignment problem. The approach proportion is defined as the proportion of passengers leaving a node through its outgoing link. To solve the VI problem, an extragradient method with adaptive stepsizes is developed. Unlike the existing methods for solving the frequency-based transit assignment problem, the convergence of our method requires only the pseudomonotone and Lipschitz continuous properties of the mapping function in VI, and it is not necessary for the Lipschitz constant to be known in advance. A Braess-like paradox in transit assignment is also discussed, where providing new lines to a transit network or increasing the frequency of an existing line may not improve the system performance in terms of expected total system travel cost. Various numerical examples are given to illustrate some paradox phenomena and to test the performance of our proposed algorithm.

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1. Introduction

The transit assignment problem has received considerable attention, as finding solutions for this issue is essential both for designing or managing transit networks and for evaluating transit system performance. Many transit assignment models have been developed, and some of the earliest can be traced back to Dial (1967) and Fearnside and Draper (1971). However, these early models did not consider the route choice problem of passengers at stops served by several competing lines—known as the common line problem.

The first model to handle the common line problem was developed by Chriqui and Robillard (1975). By assuming that passengers are willing to minimise their individual expected travel cost, a hyperbolic model was solved through finding an optimal set of attractive lines directly serving two locations. This idea was further generalised to the optimal strategy concept (Spiess, 1984; Spiess and Florian, 1989). According to this concept, a strategy associated with a node is defined by a set of rules that, when applied, allow a passenger to travel efficiently from that node to his/her destination. A strategy specifies a set of attractive lines at every node, and hence an ordered set of successor nodes. An optimal strategy is defined as a strategy that minimises the passenger's expected travel time. The behavioural assumption used by Spiess (1984) and Spiess and Florian (1989) was that passengers use their individual optimal strategies in travelling. Assuming that this is true, Spiess and Florian (1989) proposed a linear programming model to tackle the common line problem, and provided their proof that their model's dual solution satisfied the user equilibrium conditions.

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Later, two modelling streams were derived from the abovementioned behavioural assumption by using different network representations: the hyperpath graph representation (Nguyen and Pallottino, 1988; Wu et al., 1994; Cominetti and Correa, 2001; Cortés et al., 2013), and the route-section representation (de Cea and Fernández, 1993; Lam et al., 1999; Li et al., 2009; Lam et al., 2002; Szeto et al., 2011, Szeto et al., 2013). The hyperpath graph representation captures a strategy by using a hyperpath. The route-section representation aggregates common lines into sections, and a sequence of sections forms a route. Hence, each route in the route-section approach can be seen as a special case of travel strategy.

Although both of these modelling representations are based on the same behavioural assumption, they have different pros and cons. The merit of the hyperpath graph representation is that the optimal sets of attractive lines can be easily determined, but the cost involved is the requirement to create more boarding and alighting nodes. The route-section representation always reduces the numbers of links required to form the network when the number of common lines is large. Moreover, this representation allows the transit assignment problem to be easily solved by using available algorithms. However, the optimal set of attractive lines on each section has to be determined before aggregating attractive common lines.

These two modelling streams adopt similar methods to handle the in-vehicle congestion issue. These methods can be classified into two approaches, namely the capacity constraint approach and the congestion cost function approach. The difference between these approaches is that passengers are forbidden to board a fully occupied transit vehicle in the capacity constraint approach, but they are permitted to do so in the congestion cost function approach. The capacity constraint approach is more realistic, but its resultant models are normally solved by the method of successive averages (MSA), which can guarantee convergence only under some conditions, and these conditions for convergence may not be satisfied by the models themselves. In addition, there may be no solution for the problems that result from insufficient capacity. One advantage of the congestion cost function approach is that a solution must exist to the resultant transit assignment problem under a very mild assumption (*e.g.*, Szeto et al., 2013). However, the models that result from this approach may allow link flows to be greater than the link capacities, which is unrealistic.

Most of the abovementioned models are developed from the frequency-based approach, in which frequency is assumed to follow certain distributions to approximate the average waiting and travel times. However, during the last 20 years a schedule-based approach has been proposed for modelling detailed arrivals and departures (Tong and Wong, 1999; Poon et al., 2004; Hamdouch et al., 2011; Nuzzolo et al., 2012). In general, the frequency-based approach is more suitable for strategic long-term planning, and the schedule-based approach can best be used for modelling daily operations.

Most of the abovementioned models have adopted link flows, path flows or hyperpath flows as decision variables. When congestion effects are considered, the resultant models are often solved by methods that require strong conditions to be satisfied for guaranteeing convergence. For example, the symmetric linear method (*e.g.*, Wu et al., 1994) requires that the link cost function must be strictly monotone[1] for convergence. The methods of de Cea and Fernández (1993) (*i.e.*, the diagonalisation method) and of Szeto et al. (2013) (*i.e.*, self-adaptive projection and contraction algorithm with column generation) have assumed monotone[2] mapping to ensure convergence. Kurauchi et al. (2003), Cepeda et al. (2006), Sumalee et al. (2009), Sumalee et al. (2010), Schmöcker et al. (2011), and Cortés et al. (2013) all adopted the MSA, whose convergence requires the cost function to be strictly pseudo-contractive (Johnson, 1972). However, these conditions may not always be satisfied, especially when asymmetric link cost functions are used in transit assignment.

This paper proposes a link-based variational inequality (VI) formulation, which can be transformed into an approach-based formulation. This proposed formulation is based on the concept of approach proportion. To solve the approach-based formulation, we use an extragradient method, also known as the double projection method. The convergence of the algorithm only requires mild assumptions, *i.e.*, pseudomonotone[3] and Lipschitz continuous properties. Moreover, it is not necessary to know the Lipschitz constant in advance. This algorithm, however, cannot be used directly to solve our proposed formulation, and some modifications of the cost and flow updating algorithms are required. Hence, we also propose a cost and a flow updating scheme.

The proposed formulation can also be used to evaluate the performance of a transit network design. In fact, one important application of any transit assignment model is to evaluate network design strategies, such as proposals to improve the system performance through new transit itineraries or adjustments to service frequency. To the best of our knowledge, little effort has been spent on investigating whether a paradox actually exists in transit assignment, even though there are many studies concerning paradoxes in traffic assignment, and there are similarities between traffic assignment and transit assignment in terms of formulation approaches. Only Cominetti and Correa (2001) have actually revealed a paradox, showing that the transit time will not be affected under a certain range of demand increments. However, unlike Cominetti and Correa's work, in which the changes occur at the demand side, our paper focuses on the changes in the supply side of transit networks, such as routes and frequencies. A small network is created, and various scenarios are tested to investigate the existence of the paradox and the roles that different parameters or factors play in its occurrence. Our numerical results verify that providing a new transit line and increasing transit frequency may fail to improve, and may even deteriorate the network performance in terms of expected total system travel cost (including in-vehicle travel time cost and waiting time cost).

The contributions of this paper include the following:

[1] A vector function $\boldsymbol{F}$ is strictly monotone on a non-empty set C if for all $\mathbf{x}, \mathbf{y} \in C$, $\mathbf{x} \neq \mathbf{y}$, $(\mathbf{y}-\mathbf{x})^T(\mathbf{F}(\mathbf{y})-F(\mathbf{x})) > 0$.

[2] A vector function $\boldsymbol{F}$ is monotone on a non-empty set C if for all $\mathbf{x}, \mathbf{y} \in C$, $\mathbf{x} \neq \mathbf{y}$, $(\mathbf{y}-\mathbf{x})^T(\mathbf{F}(\mathbf{y})-\mathbf{F}(\mathbf{x})) \geqslant 0$.

[3] A vector function $\boldsymbol{F}$ is pseudomonotone on a non-empty set C if for all $\mathbf{x}, \mathbf{y} \in C$, $(\mathbf{y}-\mathbf{x})^T\mathbf{F}(\mathbf{x}) \geqslant 0$ implies that $(\mathbf{y}-\mathbf{x})^T\mathbf{F}(\mathbf{y}) \geqslant 0$.

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