Transportation Research Part B 68 (2014) 239–261

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Non-planar hole-generated networks and link flow observability based on link counters

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ARTICLE INFO

Article history:
Received 8 January 2014
Received in revised form 23 June 2014
Accepted 24 June 2014

Keywords:
Flow estimation problem
Optimal sensor location
Planar and non-planar networks

ABSTRACT

The concepts of hole, cycle added link and non-planar hole-generated network are introduced for the first time and used to determine (a) the immediate solution of the node conservation equations in terms of hole and cycle added vectors, and (b) the paths as linear combinations of hole vectors. Two equivalent formulas to obtain the number of links to be observed for complete link observability in non-planar hole-generated networks are given in terms of the numbers of links, nodes, holes, cycle added links and centroid node types. These formulas are applicable without any limitation in the number of centroids and possible link connections. Some simple methods are given to obtain first the maximum number of linearly independent (l.i.) paths and next a minimum set of links to be counted in order to get observability of all link flows. It is demonstrated that the number of l.i. paths in a non-planar hole-generated network coincides with the number of holes and cycle added links in the network and that any path can be obtained by linear combinations of the vectors associated with the hole and cycle added links. The methods are illustrated by their application to several networks.

1. Introduction

One of the most common problems in traffic networks consists of estimating different types of network flows, such as, path, OD or link flows. For example, Bianco et al. (2001), Cascetta and Nguyen (1988), Cascetta and Postorino (2001) and Gan et al. (2005)) discuss the OD matrix estimation problem using traffic counts. These data are very relevant to build traffic models and specially to check traffic assignment models (see, for example, Szeto et al. (2011) or Sumalee et al. (2011)).

We say that a given flow (path, OD, link, node, etc.) is observable when it can be obtained based on direct observations. Therefore the observability problem deals with determining which subset of flows are observable (can be obtained) based on the knowledge of another subset of flows.

A special observability problem is the case of link flows, which are needed for planning management strategies. To this end, knowledge of all link flows is required, but installing sensors in all links is unnecessary and very costly as some link flows depend linearly on other link flows. Thus, research has been carried out to decide which link flows need to be measured and which links can be calculated from them. In this paper we deal with the problem of determining minimal sets of links that allow full observability of all link flows. More precisely, we deal with the link observability problem based on link flow observations provided by counters (see Hu et al. (2009) or Castillo et al. (2013b)).

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http://dx.doi.org/10.1016/j.trb.2014.06.015
0191-2615/

Notation

b_i	number of columns eliminated before iteration i
c	number of output and input node flows
$\mathbf{d}$	vector of link capacities
$\mathbf{e}$	set of output minus input node flows
$\mathbf{f}$	path flow vector
$\mathbf{f}_0$	subset of vector $\mathbf{f}$
h	number of finite (bounded) holes
h_0	number of holes
k	number of added links
ℓ	number of links
ℓ^*	number of links of the maximal planar network associated with a given network
$\bar{n}$	number of no-centroid nodes
n	number of nodes
n_p	number of paths
p	path
$\mathbf{p}$	path vector
p_j	path vector
q_i	number of links used by the set of paths $\mathcal{S}_{i-1}$
q_{np}	number of flows to be observed for full link observability in non-planar networks
q_p	number of flows to be observed for full link observability in planar networks
r	number of linearly independent paths
r_j	flow of route j
$rank(\mathbf{B})$	rank of matrix $\mathbf{B}$
t_i	number of eliminated rows before iteration i
u_i	i plus the number of all other paths that use only links in any of the paths in $\mathcal{S}_{i-1}$
$\mathbf{w}_j$	edges of the cone
A_j	vector associated with the cycle added link j
$\mathbf{A}_\pi$	cone generated by the columns of matrix $\mathbf{A}$
A	link-path incidence matrix
$\mathbf{B}$	node-link incidence matrix
$\mathbf{B}^*$	node-link incidence matrix of the associated maximal planar network
E^n	Euclidean space of dimension n
H_j	hole
$\mathbf{I}$	identity matrix
$\mathbf{K}$	hole and added vector matrix
$\mathbf{K}_j$	subset of matrix $\mathbf{K}$
$\mathcal{N}$	network
$\mathcal{N}^*$	no-centroid network associated with a given network $\mathcal{N}$
$N(A)$	null space of matrix A
$\mathcal{P}$	set of paths
$\mathcal{R}_p$	set of links in path p
$\mathcal{S}$	subset of selected linearly independent paths
$\mathcal{S}_i$	set $\mathcal{S}$ at stage i
$\mathbf{U}$	matrix of constant coefficients containing in its columns the polytope vertices
$\boldsymbol{W}$	matrix whose columns coincide with all the topological feasible paths
$\mathbf{W}_0$	subset of matrix $\mathbf{W}$
α	non-negative real number in the range $(0,1)$
$\boldsymbol{\lambda}$	column vector of non-negative real numbers adding to one
v_j	flow on link j
$\boldsymbol{v}$	link flow vector
$\boldsymbol{v}_1, \boldsymbol{v}_2$	subsets of vector $\boldsymbol{v}$
π	non-negative real number
$\boldsymbol{\rho}$	column vector of real numbers.

In the following we explain how the observability problem has evolved with time.

1. *Antecedents of observability problems in other fields.* The observability problem has been examined in other areas outside the transportation field such as in the field of power systems (see Mori and Tsuzuki (1991) and Conejo et al. (2007)), or water supply (see Rahal (1995) and Kumar et al. (2008)). Though the problems dealt with in these papers (water and power