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Global optimization method for network design problem with stochastic user equilibrium

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A B S T R A C T

In this paper, we consider the continuous road network design problem with stochastic user equilibrium constraint that aims to optimize the network performance via road capacity expansion. The network flow pattern is subject to stochastic user equilibrium, specifically, the logit route choice model. The resulting formulation, a nonlinear nonconvex programming problem, is firstly transformed into a nonlinear program with only logarithmic functions as nonlinear terms, for which a tight linear programming relaxation is derived by using an outer-approximation technique. The linear programming relaxation is then embedded within a global optimization solution algorithm based on range reduction technique, and the proposed approach is proved to converge to a global optimum.

1. Introduction

The continuous network design problem (CNDP) deals with determining the link capacity expansions to achieve a certain optimal objective, subject to the resultant flow pattern following some sort of equilibrium conditions. The objective is typically taken to be the sum of the total travel time cost and the investment cost for link capacity expansions. The decision on link capacity expansions affects the resultant flow pattern, which can be described by, for example, the principle of deterministic user equilibrium (DUE) or stochastic user equilibrium (SUE) (Sheffi, 1985). Traditionally, the CNDP is formulated as a mathematical program with equilibrium constraints.

There is a vast volume of literature published on the topic of continuous network design. Yang and Bell (1998) and Farahani et al. (2013) presented comprehensive surveys on models and algorithms for this problem. Abdulaal and LeBlanc (1979) proposed the Hooke–Jeeves algorithm to solve the continuous network design problem (CNDP). Tan et al. (1979) represented the DUE problem by a set of nonlinear, non-convex inequalities. In addition, a number of sensitivity-based heuristic algorithms were developed to solve the CNDP and relevant problems (Cho, 1988; Friesz et al., 1990; Yang, 1997; Yang and Yagar, 1995). Furthermore, Suwansirikul et al. (1987) proposed an alternative heuristic to solve the CNDP, referred to as the equilibrium decomposed optimization (EDO). Meng et al. (2001) transformed the bi-level program to a single-level continuously differentiable optimization problem and employed the augmented Lagrangian algorithm to find an exact local solution. Besides, in some recent researches on network design problem, symbiotic relationship among coexisting networks are considered in modelling transport network design (Chow and Sayarshad, 2014), activity-based network design models are proposed based on the assumption that OD demand is subject of responses in household or user itinerary choices to infrastructure improvements (Kang et al., 2013).

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Nevertheless, few studies in the literature dealt with the network design problem with stochastic user equilibrium travel pattern. Davis (1994) exploited standard nonlinear programming solution methods to find the exact local optimum of the network design problem with stochastic user equilibrium. Due to the inherent nonconvex property of the network design problem formulation, it is very hard to find the global optimal solution. More recently, Wang and Lo (2010) proposed a path-based algorithm seeking the global optimal solution of the CNDP by approximating the problem as a mixed-integer linear programming (MILP) problem. Luathep et al. (2011) proposed a link-based global optimization algorithm for solving a mixed transportation network design problem, in which a series of MILPs need to be solved in iterations and a cutting constraint algorithm is applied. Li et al. (2012) developed a global optimization method based on the concepts of gap function and penalty for continuous network design problem, which is then reformulated to a sequence of concave programming subproblems. A multicutting plane method is used for solving each sub-problems. Wang et al. (2013) designed two global optimization methods for discrete network design problem, both of them taking advantage of the relationship between user equilibrium and system optimal traffic assignment and one differing the other by adding a convex constraint to tighten the system optimal model relaxation. The approach of using MILP to approximate the nonlinear functions was also studied in other traffic problems with equilibrium constraints, like toll design problems (Ekström et al., 2013; Ekström et al., 2012; Zhang and van Wee, 2012). Ekström et al. (2012) applied an MILP method that is similar with Luathep et al. (2011) to find the optimal toll locations of a toll design problem with the objective to minimize the total travel time and the cost of locating a toll. Zhang and van Wee (2012) extended the method of Wang and Lo (2010) on a congestion pricing problem, which adopted a multiplier as its objective function to maximize the reserve capacity of network. Considering that the solution algorithm in Luathep et al. (2011) need to solve a series of MILP iteratively because of the cutting constraint algorithm, Ekström et al. (2013) developed this method by incorporating a branch-and-cut algorithm and thus only one MILP is need to be solved; besides, they also considered elastic travel demand and a domain reduction scheme in their paper. Furthermore, a benders decomposition method (Fontaine and Minner, 2014) was proposed for solving discrete–continuous linear bilevel problems in the context of transport network design. In this study, we consider the continuous network design problem with SUE, specifically, logit route choice model, and propose a global optimization algorithm to obtain the global optimum of the problem. The CNDP with SUE is investigated not only because it is more realistic in describing route choice behaviour, but also because SUE model is a more general statement of user equilibrium, which includes deterministic user equilibrium conditions as a special case. With the original nonlinear and nonconvex programming formulation, we firstly take logarithm of the nonlinear terms, including multi-variable monomials describing the travel time/cost functions and the logit model. By doing so, we transform the nonlinearity of the original problem into the general concave logarithmic functions only. Then, an outer approximation technique is applied to construct a very tight linear programming relaxation. Specifically, by selecting break points to discretize the feasible domain of each logarithmic function, we use the associated tangential supports on the breakpoints, as well as chord lines connecting neighbouring break points, formulated by mixed-integer piecewise linearization, as the outer approximation of logarithmic function. Thus, a mixed-integer linear programming relaxation is derived, whose solution provides a lower bound of the original problem. The linear programming relaxation is then embedded within a global optimization algorithm based on a range reduction technique. The proposed approach is proved to converge to a global optimum.

The solution method proposed in this study is able to obtain the exact global optimum of the continuous network design problem with stochastic user equilibrium. The key lies in the nonlinearity transformation from the multivariable nonconvex functions in the original formulation to general single-variable concave logarithmic functions. By constructing an outer approximation for the concave logarithmic function, we can derive a linear programming relaxation, whose solution is much easier to be obtained and provide lower bound of the original problem. It should be noted that the linear programming relaxation, formulated as a mixed-integer linear programming, is a very tight relaxation, and its solution will be sufficiently close to the global optimum if the discretization scheme applied is fine enough, which is exactly the spirit of global optimization solution methods to CNDP developed in some previous studies (e.g., Wang and Lo, 2010; Luathep et al., 2011). However, the obtained solutions in these studies are only optimum of the linearized approximation of the original CNDP, rather than the exact solution to the original problem. In this study, the mixed-integer linear program is developed as a relaxation problem of the original model formulation. With the solution of the mixed-integer linear programming relaxation as the lower bound, a range reduction technique is used to propose an iterative solution algorithm in this study, ensuring the exact global optimality of the solution to the original problem. Similar range reduction technique is also employed in Ekström et al. (2013) as a pre-processing scheme, which updated variable bounds only once before the branch-and-cut process and thus the MILP approximation of the original nonlinear problem still relies on large number of breakpoints. In this paper, the range reduction will be embedded in an iterative process, thus the tightening of the relaxation model largely relies on the reduction of the variable bounds and rather smaller number of breakpoints is needed than the methods developed in previous paper.

The proposing method in this paper can also be applied to solve network design problems with DUE by either employing SUE model to approximate DUE via a large scaling parameter, or by adding additional binary variables and extra constraints to guarantee strictly positive value of variables in the logarithmic functions. For illustration purpose, we only elaborate continuous network design problem with SUE in this paper.

The remainder of this paper is organized as follows: Section 2 presents the model formulation and reformulation, Section 3 describes the global optimization solution algorithm, and Section 4 illustrates the computational study. Section 5 concludes the paper with a summary.