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Optimization models for differentiating quality of service levels in probabilistic network capacity design problems

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ABSTRACT

This paper develops various chance-constrained models for optimizing the probabilistic network design problem (PNDP), where we differentiate the quality of service (QoS) and measure the related network performance under uncertain demand. The upper level problem of PNDP designs continuous/discrete link capacities shared by multi-commodity flows, and the lower level problem differentiates the corresponding QoS for demand satisfaction, to prioritize customers and/or commodities. We consider PNDP variants that have either fixed flows (formulated at the upper level) or recourse flows (at the lower level) according to different applications. We transform each probabilistic model into a mixed-integer program, and derive polynomial-time algorithms for special cases with single-row chance constraints. The paper formulates benchmark stochastic programming models by either enforcing to meet all demand or penalizing unmet demand via a linear penalty function. We compare different models and approaches by testing randomly generated network instances and an instance built on the Sioux–Falls network. Numerical results demonstrate the computational efficacy of the solution approaches and derive managerial insights.

1. Introduction

Network design problems (NDPs) are essential and fundamental for the development of modern societies. Related crucial decisions include (i) an allocation of limited resources to expand existing link capacities and/or to add new links, (ii) an assignment of origin–destination (OD) pairs, and (iii) a plan of static/dynamic network flows. The decisions are often made under uncertainty due to demand variation and random network disruptions. Therefore, the objective of stochastic NDPs takes into account probabilistic measures of specific network performance indexes, such as the expected generalized cost, travel time, and demand satisfaction rate.

Depending on the nature of capacity design variables, NDPs in general fall into three categories with continuous, discrete (usually 0–1 binary), and mixed decision variables at the design phase, or equivalently speaking, at the *upper* level. For stochastic NDPs, the capacity design decisions are made before knowing the uncertainty, and additional recourse decisions are determined according to specific realizations of uncertain parameter, at the *lower* level. For specific applications, flow decisions between OD pairs may need to be determined *a priori* to the realization of the uncertainty, or could be adjustable so as to maximize the expected profit associated with some network performance index. Respectively, they are either decisions made at the upper level or recourse variables made at the lower level.

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1.1. Previous work on network design problems

NDPs have been intensively studied for transportation networks, of which a general formulation determines expansion or addition of links at the upper level, subject to user equilibrium constraints that reflect routing choice behavior of travelers at the lower level. Wang et al. (2013), Li et al. (2012), Gao et al. (2005), and Luathep et al. (2011) apply global optimization methods and meta-heuristic approaches to study discrete, continuous, and mixed NDPs. Stochastic programming (Patil and Ukkusuri, 2007; Ukkusuri and Waller, 2008) and robust optimization methods (Ukkusuri et al., 2006; Chung et al., 2011; Lou et al., 2009; Yin et al., 2009) have been developed for optimizing NDPs for unknown demand, among which the papers by Ukkusuri and Waller (2008) and Chung et al. (2011) also incorporate problem dynamics into their stochastic NDP studies. Moreover, Sharma et al. (2010) use sampling-based approximation algorithms for computing large-scale non-convex NDP problems. Ukkusuri and Patil (2009) consider multi-time-period network investment decisions as compared to a single-stage capacity expansion, and formulate the corresponding NDP variant as a bi-level stochastic program with complementarity constraints. Overall, Chen et al. (2011) provide a comprehensive review of formulations and methodologies for solving transport NDPs under uncertainty.

The NDP arises in a broader class of network-related applications, including supply chain management (MirHassani et al., 2000; Santoso et al., 2005), emergency response (Chang et al., 2007; Oh and Haghani, 1997; Sheu, 2007), and shelter relief (Kulshrestha et al., 2011), where the upper-level problem determines continuous or discrete capacity levels, followed by the lower-level problem having flow recourses. The literature often formulates two-stage stochastic optimization models and employs sampling techniques, approximation algorithms and/or cutting-plane methods, such as the Benders cuts (see, e.g., MirHassani et al., 2000; Barton et al., 1989), for iteratively optimizing the design and flow solutions.

In general, the NDP has various types, including transport NDPs involving traffic equilibria (if exist) and other NDPs only concerning flow balances between OD pairs. The former usually involve a traffic assignment problem, where individual users' flow decisions are taken into account and use equilibrium constraints are formulated in the mathematical models involved, as each user works to reduce his or her own transport costs given a designed network. Patriksson (1994) discusses various traffic assignment models and provides comprehensive algorithms for computing equilibrium solutions. However, when there is insufficient information on the travel costs faced by individual users, it becomes difficult to determine the equilibrium flow. In some other circumstances, network flows (e.g., shipments from suppliers to customers in a supply chain network) are assigned by a central operator who also design the network and no individual users are involved. The NDPs in these cases only impose balances of single- or multi-commodity flows, but are usually coupled with randomness in the demand data. We consider the latter type of NDPs, in which we design network arcs and balance supply-to-demand flows without user equilibrium constraints.

1.2. Contributions and structure of paper

In this paper, we study probabilistic network design problems (PNDPs) under demand uncertainty and with multi-commodity flows, which can be interpreted as shipments of multiple products, or other types of heterogeneous flows involved in a wide class of applications. The flow variables obey balance constraints and knapsack constraints that limit the summation of all flows within a shared capacity on every arc (e.g., Ahuja et al., 1993). Instead of the expectation-based stochastic programming approach, we use probabilistic constraints, or chance constraints (Charnes et al., 1958) to differentiate demand satisfaction rates of shipping multiple commodities to different locations (nodes). The paper formulates four types of chance-constrained models to allow the flexibility in differentiating Quality of Service (QoS) levels with respect to commodity and node-wise demand.

Following the order of PNDP variants discussed in this paper, we determine both flow variables and continuous capacity expansion variables of existing links at the upper level, and evaluate demand satisfaction rates at the lower level. The aim is to minimize the total cost of capacity expansion and flow assignment subject to various forms of chance constraints for bounding the demand losses. We justify this approach by the application of supply chain design, where a flow scheduler needs to be decided before knowing the uncertainty and cannot be easily adjusted in different scenarios. For instance, due to high contracting fees and the ease of maintaining a relationship with a stable set of suppliers and customers, the scheduler may prefer a fixed shipping schedule regardless of actual daily demand when the demand fluctuation is not significant. In some emergency response applications, reaction time for changing the delivery plan may be too short so that fixed flows are more favorable.

We formulate benchmark stochastic network design problems (SNDPs) by letting flows be recourse variables, whose values are determined *after* knowing the demand. The modification results in the flexibility of having different flow decisions in each scenario, but enforces a hard constraint on the flow decisions. We consider two types of SNDPs, of which one that does not penalize unmet demand, and the other adds linear penalty cost for each unit of unmet demand. The latter is typically used in the existing literature and will provide benchmarks in our numerical results. Furthermore, we consider PNDPs and SNDPs with discrete (binary) design variables, representing the addition of new links, and reformulate all formulation variants under this assumption.

Our paper adopts a static modeling framework, and studies the NDP with multi-commodity flows without user equilibrium constraints. We transform all probabilistic models into equivalent mixed-integer programming (MIP) formulations under the assumption of finitely distributed random demand. For some special cases of the PNDP, we present alternative