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Transition choice probabilities and welfare analysis in random utility models with imperfect before–after correlation

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ABSTRACT

Welfare in random utility models is used to be analysed on the basis of only the expectation of the compensating variation. De Palma and Kilani (De Palma, A., Kilani, K., 2011. Transition choice probabilities and welfare analysis in additive random utility models. Economic Theory 46(3), 427–454) have developed a framework for conditional welfare analysis which provides analytic expressions of transition choice probabilities and associated welfare measures. The contribution is of practical relevance in transportation because it allows to compute shares of shifters and non-shifters and attribute benefits to them in a rigorous way. In De Palma and Kilani (2011) the usual assumption of unchanged random terms before and after is made.

The paper generalises the framework for conditional welfare analysis to cases of imperfect before–after association of the random terms. The joint before–after distribution of the random terms is introduced with postulated properties in terms of marginal distributions and covariance matrix. Analytic expressions, based on the probability density function and the cumulative distribution function of the joint before–after distribution, and simulation procedures for computation of the transition choice probabilities and the conditional expectations of the compensating variation are provided. Results are specialised for multinomial logit and probit. In the case without income effects, it is proved that the unconditional expectation of the compensating variation depends only on the marginal distributions.

The theory is illustrated by a numerical example which refers to a multinomial logit applied to the choice of the transport mode with two specifications, one without and one with income effects. Results show that transition probabilities and conditional welfare measures are affected significantly by the assumption on the before–after correlation. The variability in the transition probabilities across transitions tends to decrease as the before–after correlation decreases. In the extreme case of independent random terms, the conditional expectations of the compensating variation tend to be close to the unconditional expectation.

1. Introduction

Discrete choice models derived by assuming that each individual maximises a random utility function are used extensively to represent demand for transport alternatives. The assessment of the benefits that the choice makers receive from

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the implementation of policies which change the attributes of the choice alternatives is of significant practical importance. Several authors (De Jong et al., 2007; Jara-Díaz and Farah, 1988) have argued that the assessment of the benefits accruing to choice makers should be based rigorously on measures that are consistent with the microeconomic foundation of the models used for representing their choices. This requirement, together with the need for expressing changes in well-being in monetary terms, in order to make comparisons with the costs of the policies feasible, points to welfare as the most appropriate measure.

The mainstream approach to welfare measurement with random utility models, today, is based on random compensating variation. The compensating variation measures the welfare variation associated with a policy which changes the price and quality of the alternatives. Given that utility is treated as a random variable, it is most natural to adopt as welfare measure the expectation of the compensating variation derived from random utility.

Random utility models are commonly interpreted by regarding the random terms as individual specific. Given this interpretation, the expectation of the compensating variation is the average of the measure for a population of individuals with identical systematic part of the utilities.

The expectation of the compensating variation can be computed in simulation, i.e. by drawing from the distribution of the random terms. A few authors have derived analytic expressions which are less computationally demanding than simulation.

In the case without income effects, i.e. when income does not affect choices, the analytic solution is particularly simple and convenient. McFadden (1999) proved that the expectation of the compensating variation equals the difference, normalised by the marginal utility of income, of the expectations before and after the change of the maximum utility and takes a logsum form for logit.

There are, however, applications where the hypothesis of absence of income effects is, at least in principle, questionable. Income effects can be postulated in applications with high travel prices, which may occur for motorway tolls, charges resulting from road pricing policies, airline tickets. Franklin (2006) and Viauroux (2011) provide empirical evidence of income effects at the level of mode choice in urban and metropolitan areas. Another case is the one of developing countries where expenditure in transport can be a significant fraction of many socio-economic groups' budget, as shown by Jara-Díaz and de Dios Ortúzar (1989). Income effects imply that an individual who is facing a set of alternatives and alternative attributes may make different choices at different levels of her income.

The first approach proposed to compute the expectation of the compensating variation in the case with income effects is based on simulation (Cherchi et al., 2004; Herriges and Kling, 1999; McFadden, 1999). Later, Dagsvik and Karlström (2005) provided a formula for additive models on the basis of the distribution of the compensating variation. Using an approach based on the definition of the random expenditure function and Hicksian, i.e. compensated, probabilities, they show that the expectation of the compensating variation is expressed by a one-dimensional integral which can be computed numerically using non-simulation methods when the probabilities are available in closed form. They also provide specialisation of the result to random utility models used frequently in applied work such as multinomial and nested logit.

To compute the expectation of the compensating variation it is necessary to make an assumption on the correlation of the random terms between the state before the change and the state after the change. For reasons of mathematical convenience, all the methods developed to provide the expectation of the compensating variation analytically have assumed that the random terms remain the same before and after, i.e. an assumption of perfect before–after correlation is made. The assumption is usually retained also when the measure is computed by simulation.

Relaxing the perfect correlation assumption is justified. In the interpretation of random utility models that regards the random terms as individual specific, each individual, when faced with a sequence of choices, may change her random term vector. This change can be seen as a "move" of the individual along the distribution of the random terms which, by assumption, is unchanged. The "move" can be explained in two ways. The random terms can be regarded as deterministic to the individual but random to the modeller due to variables that are known by the individual but unobserved by the modeller. These variables may be attributes of the alternatives that are changed by the policy. Notice that the mean of these unobservables, represented in the alternative-specific constant, is assumed to be unchanged under unchanged distribution, while the individual-specific random term vector changes. Alternatively, one may regard that tastes are random to the individual herself, in the sense that they vary from one choice to the next in a way which cannot be fully predicted by the individual.

The statistical association of the random terms between the before state and the after state is, therefore, a relevant issue. In this respect, independence across choices is an un-necessarily restrictive assumption, since some degree of serial correlation is likely to exist because of temporally persistent unobservables and tastes.

Few authors have dealt to date with the impact of the before–after correlation. Zhao et al. (2012) use simulation to provide the expectation of the compensating variation in logit in the case without income effects. Before and after random terms are generated based on rank order correlation, independently and with identical correlation across alternatives. They find that the logsum formula is robust to the assumption on the before–after correlation because the differences of the logsum with the measures obtained from simulation with varying before–after correlation result to be negligible.

Zhao et al. (2012) have investigated the variation of the compensating variation within a population of individuals with identical systematic part of the utilities. They find that the variation may be large and stress that the expectation is to be