Transportation Research Part B 63 (2014) 53–76

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Waiting for public transport services: Queueing analysis with balking and reneging behaviors of impatient passengers

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ARTICLE INFO

Article history:
Received 17 February 2013
Received in revised form 18 February 2014
Accepted 18 February 2014

Keywords:
Queues of batch arrival and bulk service
Balking
Reneging
Compound Poisson processes
Bus bridging
Public transport

ABSTRACT

Queues of batch arrivals and bulk service including balking and reneging behaviors of customers are commonly observed in real life. This study formulates queues of this type using compound Poisson processes and determines some key probabilistic measures. Analytical investigation is undertaken yielding a range of mathematical results. The developed mathematical model and approaches apply to a variety of practical queueing processes that are featured with bulk queues, balking, and reneging. A bus bridging response to rail disruption is considered as an application example. And large-scale Monte-Carlo simulations are conducted to demonstrate the mathematical results.

1. Introduction

Queueing or waiting for services is one of the disagreeable but necessary experiences of life. To a large extent, queueing theory originated and has grown from the study of such experiences. There is a vast amount of human-queueing phenomena among which those associated with public transport services are commonly observed on a daily basis (Ceder, 2007; Higgins and Kozan, 1998; Huisman and Boucherie, 2001; Marguier and Ceder, 1984; Trietsch, 1993;Vansteenwegen and Van Oudheusden, 2007). This study addresses a special class of queueing problems with an orientation to public transport services.

Queueing theory characterizes queueing systems according to (see e.g. Allen, 1990; Gross et al., 2008; Kleinrock, 1975): (a) arrival patterns of customers (e.g. Poisson/Erlang/general); (b) service patterns; (c) queue discipline (e.g. first-come-first-served, priority-based); (d) the number of servers provided; (e) the maximum queue length allowed; (f) configuration of servers (e.g. in series/in parallel/mixed). This paper is primarily concerned with the arrival and service patterns, the behavior of impatient customers, and the impact of such behavior on queueing. More precisely, the focus is on queueing processes of the following features:

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[1] The major part of this work was conducted when the first author was with the Department of Civil Engineering at Monash University, Australia.

http://dx.doi.org/10.1016/j.trb.2014.02.004
0191-2615/

(i) *batch arrival*: customers arrive in teams rather than individually;
(ii) *bulk service*: customers are served by the server (or each server in the case of multiple servers) in teams rather than individually;
(iii) *balking*: some customers choose not to join a queue upon their arrivals, normally because of too long a queue ahead;
(iv) *reneging*: some other customers first choose to join a queue, but gradually lose their patience, and eventually leave the queue before receiving service in case of intolerable waiting.

Features (i) and (ii) fall in the categories of arrival and service patterns, while features (iii) and (iv) are mostly concerned with the queueing psychology.[2] Queueing of the above features are ubiquitous in the real world, and typically seen in the sectors of public services, transportation, manufacturing, and telecommunication. Although this paper concentrates on queueing of these features in public transport services, a brief survey is first presented on bulk queues with balking and reneging in a much broader sense.

Queueing with *batch arrivals* and *bulk service* are common (Claeys et al., 2011; Powell, 1985; Sikdar and Gupta, 2008). A server of a certain capacity becomes available after a random amount of time to serve a pool of customers. If the capacity is less than the number of customers waiting, the server leaves behind some customers (Kahraman and Gosavi, 2011). Elevators in buildings form a common example for this type of systems. Other examples arise in various settings. In transport and freight systems, queues of this type are found with airport buses/metros/taxis, urban buses/trains/metros/trams, people-movers (e.g. cable cars in amusement parks), cargo-delivering airplanes/ships, etc. In the setting of manufacturing, machines may serve several units at the same time. For instance, equipment for heat treatment can usually handle a number of parts simultaneously. Automated guided vehicles to deliver jobs from one site to another, which are used in both freight (e.g. harbor-related) and manufacturing settings, often involve bulk queues.[3] In the setting of information technologies, individual information packets are grouped in larger entities for transmission. In addition, the operation of online reservation systems is generally related to bulk queues. Besides the above examples concerning real-time operations, bulk queues may also be noticed over a larger scale of time (e.g. days/months), e.g. the ordering of some special goods or service. Despite a body of literature on bulk queueing systems, the classics of queueing theory (e.g. Allen, 1990; Gross et al., 2008; Kleinrock, 1975) focuses on queueing systems of single arrivals and service, and takes bulk queues as a special case. Specifically for bulk queues in public transportation systems, only limited work has been published (Powell, 1983, 1985; Rapoport et al., 2010; Sim and Templeton, 1982; Selvi, 1983).

Customers are often discouraged by long queues. They usually tend to join a queue only when a short wait is expected or first join it but depart if a further wait would be intolerable. This leads to two actions: *balking* (the refusal of an arriving customer to join a queue); *reneging* (the departure of a queueing customer before obtaining aimed service). Although the phenomenon that customers are "lost" through balking and reneging are widespread in real life, the classical queueing theory is primarily concerned with queues in which customers are all patient and eventually get served. The simplest balking phenomenon is observed in the loss system where arrivals do not enter the system when all servers are found busy. The study of the loss system can be traced back to 1917 when the Danish mathematician A. K. Erlang, a pioneer in queueing theory, considered the calls lost by a busy telephone exchange and derived the renowned Erlang's loss formula (Allen, 1990; Gross et al., 2008; Kleinrock, 1975). A balking behavior may generally depend on the queue length, while the period that a customer stays in line before reneging is usually modeled as a random variable. Readers are referred to (Al-Seedya et al., 2009; Barrer, 1957; Blackburn, 1972; Rao, 1965; Stanford, 1979; Ziya et al., 2006) for queueing theory with balking and reneging. The late renowned transport researcher Frank Haight was among the earliest group of researchers who studied reneging (Haight, 1959).

It is not rare to see queues with features (i)–(iv) in practice, e.g. queueing at the entry of a popular restaurant, where customers arrive by groups/families, served by tables, and balking and reneging certainly happen. Such queues relating to public transport services are observed at:

- an outbound bus/metro/taxis station at an airport (where passengers arrive in teams from landed flights and may be served in teams, involved with balking and reneging);
- an urban bus/train/tram/metro station;
- an entry to a people-mover (e.g. cable-car) in an amusement park.

[2] Limited attention was given to this regard (Maister, 1985; Larson, 1987). More precisely, human factors and the psychology of queueing customers played little role in queueing theory and therefore were not regarded as one specific aspect of queueing processes. Nevertheless, according to Allen (1990), Gross et al. (2008), and Kleinrock (1975), the impacts of customer psychology such as balking and reneging are attributed to the category of arrival patterns. It should also be pointed out that whenever an imposed maximum queue length is reached (see (e) in the proceeding text), balking happens to any subsequently arriving customer until after the queue length becomes lower than the maximum permissible limit.

[3] In this paper a bulk queue refers to a queue with batch arrivals and/or bulk service.