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# assignment problem from a Bayesian perspective

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#### ABSTRACT

This study proposes a formulation of the within-day dynamic stochastic traffic assignment problem. Considering the stochastic nature of route choice behavior, we treat the solution to the assignment problem as the conditional joint distribution of route traffic, given that the network is in dynamic stochastic user equilibrium. We acquire the conditional joint probability distribution using Bayes' theorem. A Metropolis–Hastings sampling scheme is developed to estimate the characteristics (e.g., mean and variance) of the route traffic. The proposed formulation has no special requirements for the traffic flow models and user behavior models, and so is easily implemented.

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## 1. Introduction

Because of the stochastic nature of user behavior, stochastic assignment models have long been recognized as an important area of study. Stochastic assignment models are consistent with the random utility theory (Daganzo and Sheffi, 1977) and are useful in evaluating the effect of user behavior on traffic flow. As a result, these models have been well documented over the past few decades. Daganzo (1977) considered that the day-to-day traffic flow fluctuations in a stochastic user equilibrium (SUE) network are due to stochastic travel behavior and that the traffic flow should have a probability distribution. Based on a user behavioral foundation employed by Daganzo and Sheffi (1977) and Hazelton et al. (1996) discussed a class of contemporaneous user interaction models. Their study found that the solution to the models should be a joint distribution of route choices. In this case, the joint distribution cannot be obtained, but the characteristics of the distribution can be estimated from route choice samples drawn from the joint distribution. In a recent work (Wei et al., 2010), the authors proposed a new formulation for the static stochastic assignment problem. The new formulation is more computationally efficient and flexible than that of Hazelton et al. (1996), because it explicitly derives the traffic flow probability distribution and directly assigns route traffic flows rather than individual route choices.

To take into account the actual traffic movement within a day, efforts have been made to develop within-day dynamic models for stochastic traffic assignment problems. Cascetta and Cantarella (1991) proposed a doubly dynamic assignment model along the same lines as the models of Daganzo (1977) and Cascetta (1989). They use a Markov chain in their model to formulate the traffic assignment problem. Although the model accounts for within-day traffic flows, it assumes that users react only to their travel experiences of the previous day. Ran and Boyce (1996), Tong and Wong (2000), Han (2003), and

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Sumalee et al. (2009) applied the same logic as Daganzo and Sheffi (1977) to formulate the dynamic stochastic assignment problem. Their models are consistent with the random utility theory, but make it difficult to gain insight into the stochastic nature of traffic flow (i.e., to obtain the probability distributions of traffic flow). Hazelton (2003) concluded that the probability distributions of traffic flow for the within-day dynamic stochastic problem are more complex than those of the static case.

This paper reports a new formulation of the dynamic stochastic traffic assignment problem, which is a further development of the prototype model constructed by Wei et al. (2010). The proposed formulation captures both the stochastic and the dynamic nature of within-day network traffic.

In this study, we focus on route assignment and employ fixed time-dependent origin-destination (OD) demands. We assume that the study time-horizon is discretized into several time intervals, and that the OD demand per time interval is separated into several departing route traffic packets (see Lo and Szeto, 2002). Following the logic of Wei et al. (2010), we treat the departing route traffic packets as random variables, and consider that the solution of the assignment problem should be *the conditional joint distribution of departing route traffic packets, given that the network is in dynamic stochastic user equilibrium (DSUE).* We will describe how Bayes' theorem can be applied to acquire the conditional joint distribution.

The remainder of the paper is organized as follows. Section 2 applies Bayes' theorem to acquire the conditional joint distribution of departing route traffic packets and analyzes the property of the proposed assignment formulation. Section 3 develops a Metropolis–Hastings sampling scheme, which is used to estimate the characteristics of the departing route traffic (e.g., the means and variances of the departing route traffic). Section 4 presents numerical examples that show the effectiveness of the proposed methods, and Section 5 concludes the paper.

### 2. Methodology

### 2.1. Definitions

We start by introducing the prototype model proposed by Wei et al. (2010). Let *I* denote the set of users, *R* the set of routes,  $\mathbf{S} = [S_1, ..., S_{|I|}]$  the vector of user states,  $\mathbf{F} = [F_1, ..., F_{|R|}]$  the random vector of route flows, **f** the value of **F**,  $\mathbf{C} = [C_1, ..., C_{|I|}]$  users' route choices, and **c** the value of **C**. Following Daganzo and Sheffi (1977) and Hazelton (1998), Definitions 1 and 2 are given by Wei et al. (2010), as follows:

**Definition 1** (*Stochastic User Behavior (SUB)*). A user displays SUB if and only if the user selects the route that he or she perceives to have maximum utility.

The value of S<sub>i</sub> is set to 1 when user i displays SUB. Note that S<sub>i</sub> denotes the status of user i, not the route chosen by user i.

**Definition 2** (*Stochastic User Equilibrium (SUE)*). A traffic network is in SUE if and only if all users display SUB. In other words, a traffic network is in SUE if and only if S = 1 (*where* 1 *is a vector of ones*).

Wei et al. (2010) considered that the traffic flow fluctuations in an SUE network are caused by stochastic travel behavior. They treated the solution to the stochastic traffic assignment as the conditional distribution of route flows, given that the traffic network is in SUE. According to Definition 2, the probability mass function of the conditional distribution can be represented as  $P(\mathbf{F} = \mathbf{f} | \mathbf{S} = \mathbf{1})$ . To obtain the probability mass function, Wei et al. (2010) first derived the likelihood of a route choice,  $P(\mathbf{S} = \mathbf{1} | \mathbf{C} = \mathbf{c})$ , using Definition 1. Then, they applied Bayes' theorem to derive the posterior probability mass function of the conditional distribution of the conditional distribution,  $P(\mathbf{F} = \mathbf{f} | \mathbf{S} = \mathbf{1})$ , from this likelihood. Finally, they obtained the probability mass function of the conditional distribution,  $P(\mathbf{F} = \mathbf{f} | \mathbf{S} = \mathbf{1})$ , from the posterior probability mass function of route choices via the relationship between  $\mathbf{c}$  and  $\mathbf{f}$ .

In this study, we introduce a within-day time horizon into the assignment problem. As such, Definitions 1 and 2 are modified as follows.

**Definition 3** (*Dynamic SUB* (*DSUB*)). A user departing at time *t* displays DSUB if and only if the user selects the route that he or she perceives to have maximum utility at the departing moment.

We use  $S_{t,i}$  to denote the state of user *i* who departs in time interval *t*, and set  $S_{t,i} = 1$  if the user displays DSUB.

**Definition 4** (*Dynamic SUE* (*DSUE*)). A traffic network is in DSUE during the time horizon from t = 1 to t = |T| if and only if the users departing at each instant of time display DSUB. In other words, a traffic network is in DSUE if and only if  $S_{t,i} = 1 \forall i \in I_t$ ,  $t \in T$ .

In Definition 4,  $I_t$  denotes the set of users who depart in time interval t, and T denotes the set of time intervals. For the time horizon, we introduce the following variables. Let  $C_t = [C_{t,1}, \ldots, C_{t,|l_t|}]$  denote the random vector of the route choices of users departing at time t,  $c_t$  the value of  $C_t$  (i.e., the route index),  $S_t = [S_{t,1}, \ldots, S_{t,|l_t|}]$  the states of users who depart at time t,  $F_t = [F_{t,1}, \ldots, F_{t,|R_t|}]$  the route traffic packets departing at time t,  $f_t$  the value of  $F_t$ ,  $U_{t,i}(r)$  the utility of route r according to user i who departs at time t, N the set of OD pairs, and  $q_{t,n}$  the time-dependent travel demand for OD pair n at time t.

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