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On the continuum approximation of the on-and-off signal control on dynamic traffic networks

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A B S T R A C T

In the modeling of traffic networks, a signalized junction is typically treated using a binary variable to model the on-and-off nature of signal operation. While accurate, the use of binary variables can cause problems when studying large networks with many intersections. Instead, the signal control can be approximated through a continuum approach where the on-and-off control variable is replaced by a continuous priority parameter. Advantages of such approximation include elimination of the need for binary variables, lower time resolution requirements, and more flexibility and robustness in a decision environment. It also resolves the issue of discontinuous travel time functions arising from the context of dynamic traffic assignment.

Despite these advantages in application, it is not clear from a theoretical point of view how accurate is such continuum approach; i.e., to what extent is this a valid approximation for the on-and-off case. The goal of this paper is to answer these basic research questions and provide further guidance for the application of such continuum signal model. In particular, by employing the *Lighthill–Whitham–Richards* model (Lighthill and Whitham, 1955; Richards, 1956) on a traffic network, we investigate the convergence of the on-and-off signal model to the continuum model in regimes of diminishing signal cycles. We also provide numerical analyses on the continuum approximation error when the signal cycles are not infinitesimal. As we explain, such convergence results and error estimates depend on the type of fundamental diagram assumed and whether or not vehicle spillback occurs to the signalized intersection in question. Finally, a traffic signal optimization problem is presented and solved which illustrates the unique advantages of applying the continuum signal model instead of the on-and-off model.

1. Introduction

Signalized intersections play a vital role in the design, management and control of urban traffic networks. These locations are often the most restrictive bottlenecks, and therefore urban traffic control strategies tend to focus on the operation of signalized intersections (Miller, 1963; Robertson and Bretherton, 1974; Shelby, 2004; Chitour and Piccoli, 2005; Guler

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and Cassidy, 2012; Gayah and Daganzo, 2012). Thus, it is imperative that we are able to accurately predict traffic dynamics at these locations, and estimate the resulting impact on a network. Fortunately, modeling these common junctions is relatively straightforward: for a given movement at an intersection, the impact of the signal on traffic dynamics is incorporated using a single binary variable. When the signal is green for the subject movement, the binary variable allocates the entirety of the exiting link's capacity to the downstream end of the subject approach. When the signal is red, the binary variable ensures that this capacity is zero.

Unfortunately, the discrete nature of this 'on-and-off' signal timing makes studying and optimizing the control parameters of these junctions rather complex, especially on large networks with many signalized intersections. Incorporating the binary traffic signal state variables in a signal optimization process usually results in mixed integer mathematical programs; examples include Improta and Cantarella (1984), Lin and Wang (2004), Lo (1999a) and Lo (1999b). For large networks, these mixed integer mathematical programs can be very difficult to solve exactly. Even when possible, the solutions require a tremendous amount of time, which makes real-time applications impossible. Realistic extensions that account for the combination of *dynamic traffic assignment* (DTA) with signal optimization, such as the so-called *dynamic user equilibrium with signal control* (DUESC) problems, become especially difficult (Aziz and Ukkusuri, 2012; Ukkusuri et al., 2013).

To simplify the modeling of signalized intersections in networks, recent studies have proposed an elegant continuum model to approximate traffic dynamics at traffic signals in an intuitive way (Smith, 2010; Ge and Zhou, 2012). This model works as follows. Consider a simple merge junction with two incoming links, I_1 and I_2, and one outgoing link, I_3, as depicted in Fig. 1. Assume now that the junction A is controlled by a fixed-cycle traffic signal which controls the movement of the exit flows on links I_1 and I_2. The receiving capacity of the outgoing link I_3 is assumed to be time-dependent and given by the supply function $S_3(t)$. Additionally, the fraction of the cycle dedicated to link I_1 is given by η, and the fraction dedicated to link I_2 is $1-\eta$ for some $\eta \in (0,\ 1)$. The continuum model asserts that a proportion η and $1-\eta$ of the downstream link capacity $S_3(t)$ is assigned to link I_1 and I_2, respectively, during the entirety of the signal cycle. A more detailed and formal definition of such model will be provided later in Section 2.

While this continuum model will not predict traffic dynamics at the intersection exactly, it does have a number of advantages when compared to the on-and-off signal model that is typically used:

- In a discrete-time setting, the binary representation of signal control strategies will be replaced with a real-valued parameter η. This eliminates the need of using binary variables for the signalization, and significantly reduces the computational burden of the mixed integer programs such as those reviewed above.
- The on-and-off signal model usually demands a very fine time resolution to accommodate certain signal splits. For example, a cycle with 35 s of green phase and 25 s of red phase requires a time step of at most 5 s to be properly implemented. These fine time resolutions increase the computational requirements of the network simulations and/or optimizations. On the other hand such constraints do not apply to the continuum case, thus one has more flexibility in choosing the time step for computational convenience and efficiency.
- For any fixed-cycle traffic signal optimization problem defined on a prescribed time grid, the on-and-off signal strategy can only take on several discrete values, while the continuum model yields a continuous spectrum of choices and outcomes.
- The on-and-off signal control naturally results in discontinuities in travel time functions, which poses difficulties in quite a few dynamic traffic assignment models. For example, a dynamic user equilibrium problem (Friesz et al., 2013) cannot be properly defined with the on-and-off signal controls unless some sort of indifference of drivers in travel time is introduced (Szeto and Lo, 2006; Ge and Zhou, 2012; Han, 2013). Such obstacle can be easily avoided by the continuum signal model.

Despite the appealing features of the continuum signal model mentioned above, the model has never been rigorously analyzed in connection with its counterpart, the on-and-off model. From an application point of view, it is of fundamental importance to identify circumstances where such continuum approximation accurately describes the aggregate behavior that exists at signalized intersections and, perhaps more importantly, to identify when it is invalid and may induce significant error. It is also worthwhile to investigate to what extent the continuum model is a good approximation of the on-and-off signal control. Solving these objectives can help to identify situations where this approximation can be used, and when its advantages can be realized without sacrificing model accuracy. These issues serve as the motivation of the current paper and are fully addressed by the findings made herein.

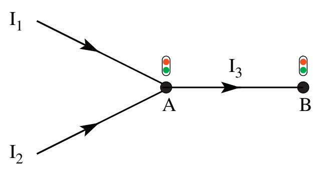

Fig. 1. A signalized merge junction.