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Total unimodularity and decomposition method for large-scale air traffic cell transmission model

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ABSTRACT

In an earlier work, Sun and Bayen built a Large-Capacity Cell Transmission Model for air traffic flow management. They formulated an integer programming problem of minimizing the total travel time of flights in the National Airspace System of the United States subject to sector capacity constraints. The integer program was relaxed to a linear program for computational efficiency. In this paper the authors formulate the optimization problem in a standard linear programming form. We analyze the total unimodular property of the constraint matrix, and prove that the linear programming relaxation generates an optimal integral solution for the original integer program. It is guaranteed to be optimal and integral if solved by a simplex related method. In order to speed up the computation, we apply the Dantzig–Wolfe Decomposition algorithm, which is shown to preserve the total unimodularity of the constraint matrix. Finally, we evaluate the performances of Sun and Bayen's relaxation solved by the interior point method and our decomposition algorithm with large-scale air traffic data.

1. Introduction

The *National Airspace System* (NAS) in the United States is a large-scale, nonlinear dynamic system. The airspace is divided into 22 *Air Route Traffic Control Centers* (ARTCCs, or simply, Centers). Each Center is sub-divided into smaller regions, called *Sectors*, with at least one air traffic controller responsible for each of them (Nolan, 2003).

The last few decades have witnessed the tremendous growth of air traffic. Since the function of air traffic controllers is to maintain safe separation between aircraft while guiding them to destinations, an imbalance between the growth of air traffic and the limited airspace capacity arises. So the design of advanced air traffic management schemes is desired to help.

Optimization techniques have been developed to facilitate *Traffic Flow Management* (TFM). Current popular TFM schemes mainly focus on ground delay and/or rerouting flights to accommodate capacitated elements, e.g., en route sectors and airports (Lulli and Odoni, 2007). TFM studies focusing on optimal ground delays have been conducted by many researchers, from both deterministic and probabilistic perspectives (see Odoni (1987), Terrab and Odoni (1993), Gilbo (1993), Vranas et al. (1994), Andreatta et al. (1997), Navazio and Romanin-Jacur (1998), Bertsimas and Patterson (1998), Hoffman and Ball (2000), Dell'Olmo and Lulli (2003), Vossen et al. (2003), Ball and Lulli (2004), Ball et al. (2005), Vossen and Ball (2005, 2006)). Odoni (1987) formulated the TFM problem using a large number of models and algorithms to detect optimal strategies to assign ground delays to flights (see in particular Bertsimas et al. (2008)). Helme (1992) was among the first to include en route capacity restrictions in the TFM problem, which is intuitive to understand but has weak computational performance as was discussed by Bertsimas et al. (2008). Lindsay et al. (1993) formulated a disaggregate deterministic 0–1 integer

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programming model for deciding ground and airborne holding of individual flights in the presence of both airport and airspace capacity constraints. A deterministic, open-loop integer programming method was formulated to assign departure time and sector occupancy time of each aircraft in the work by Bertsimas and Patterson (1998), but the computational complexity of this model has limited its use to a small number of real-world examples as was shown by Grabbe et al. (2007). To improve the runtime, a method that reduces the number of flights to be optimized was proposed by Rios and Ross (2008); and more recently, a Dantzig–Wolfe Decomposition method was implemented for the Bertsimas and Patterson model (Rios and Ross, 2010), which actually motivated several studies (including the work in this paper) using decomposition methods to solve large-scale TFM problems. The work in Bertsimas and Patterson (1998) was extended to provide a complete representation of all the phases of each flights including rerouting strategies (Bertsimas et al., 2008). In Sherali et al. (2002), a binary integer programming was proposed for a TFM problem, which considers controller workload, airspace safety, and equity among airlines. Subsequently, the binary integer programming was extended to incorporate rerouting in Sherali et al. (2003, 2006). Research that considers equity or market-based traffic management using aggregate models has been conducted by Bloem and Sridhar (2008), Waslander et al. (2008a,b). Sridhar et al. (2002) proposed an integrated three-step hierarchical method for developing deterministic TFM plans consisting of national-level playbook reroutes, miles-in-trail restrictions, and tactical reroutes to alleviate sector-level congestion. Subsequently, Kopardekar and Green (2005) used a deterministic, Center-based system to manually identify congested sectors and compare the trade-offs of implementing altitude capping, local rerouting, departure delays, and time-based metering or miles-in-trail restrictions. Wanke and Greenbaum (2007) proposed a Monte Carlo-based incremental, probabilistic decision making approach for developing en route traffic management controls. More recently, Grabbe et al. (2009) applied a sequential optimization method to manage air traffic flow under the uncertainties in airspace capacity and demand.

Sun and Bayen (2008) presented a traffic flow model called the *Large-Capacity Cell Transmission Model*, in short CTM(L), which is a variation of the air traffic cell transmission model in Menon et al. (2002) and the original cell transmission model in Daganzo (1994, 1995). Sun and Bayen applied it to a problem of minimizing the total travel time of all flights in the NAS of the United States restricted by sector capacity counts, which is an integer program containing billions of variables and constraints. It was then relaxed to a *linear program* (LP) for computational efficiency. Sun et al. (2011) applied the dual decomposition method to solve the large scale linear program in a computationally tractable manner. However, the authors in Sun and Bayen (2008) and Sun et al. (2011) found that solving the linear program by large-scale commercial software with or without decomposition method can possibly result in the fractional optimal solution, which cannot be implemented as en route holding control in practice. Integer solutions should be guaranteed while the optimum is obtained efficiently. This is the major motivation of our work.

In this paper we study the solution space structure of the problem and prove that there exists an optimal integral solution in the linear programming relaxation, which is also the optimal for the original integer program. The solution is guaranteed to be integral when solved by simplex related methods. Therefore we propose the simplex based Dantzig–Wolfe Decomposition to ensure the integral optimum, while achieving a fast computation speed.

The rest of this paper is organized as follows. The second section introduces the CTM(L) model. The third section formulates the integer programming problem in a standard linear programming form and analyzes its total unimodularity. The fourth section explains why the interior point method applied in Sun and Bayen (2008) results in the fractional optimal solution. In Section 5 we apply the Dantzig–Wolfe Decomposition algorithm. Large-scale simulations are performed with historical data. Section 6 concludes the paper.

2. CTM(L) and its mathematical formulation

The CTM(L) is based on a network flow model built from the historical *Aircraft Situation Display to Industry* (ASDI) and *Enhanced Traffic Management System* (ETMS) data (Bayen et al., 2006).

2.1. Construction of the network

The network flow model is composed of nodes and links. The *nodes* are created as the entry and exit points at the sector boundaries as shown in Fig. 1. For any sectors s_1, s_2 and s_3, if s_1 and s_2 share a boundary and if s_2 and s_3 are neighbors, two directed *links* are created: one from node $v_{\{s_1,s_2\}}$ to node $v_{\{s_2,s_3\}}$ and one from node $v_{\{s_3,s_2\}}$ to node $v_{\{s_2,s_1\}}$.

The expected travel time of a flight through a link is computed from ASDI/ETMS data, which is used to determine the length of the link. Each link is divided into several *cells* as time interval units (see Fig. 1). A *path* is defined as a complete flight route connecting one departure airport and one arrival airport, which usually consists of multiple links. Further details on constructing the CTM(L) network are described in Sun and Bayen (2008).

2.2. Dynamics

The CTM(L) model is reduced to a linear time-invariant dynamical system. The air traffic flow on link i can be depicted as the *Link Level Model* (Sun and Bayen, 2008):