Transportation Research Part B 48 (2013) 67–87

Contents lists available at SciVerse ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Risk-neutral second best toll pricing

Xuegang (Jeff) Ban [a,*], Michael C. Ferris [b], Lisa Tang [c], Shu Lu [d]

[a] Department of Civil and Environmental Engineering, Rensselaer Polytechnic Institute, Troy, NY 12180, United States
[b] Computer Sciences Department, University of Wisconsin at Madison, Madison, WI 53706, United States
[c] Industrial and Systems Engineering Department, University of Wisconsin at Madison, Madison, WI 53706, United States
[d] Department of Statistics and Operations Research, University of North Carolina at Chapel Hill, Chapel Hill, NC 27599, United States

A R T I C L E I N F O

Article history:
Received 5 August 2009
Received in revised form 30 November 2012
Accepted 30 November 2012

Keywords:
Congestion pricing
Second best toll pricing
Risk-neutral
Traffic user equilibrium
Stochastic program
Simulation based optimization

A B S T R A C T

We propose a risk-neutral second best toll pricing (SBTP) scheme to account for the possible nonuniqueness of user equilibrium solutions. The scheme is designed to optimize for the expected objective value as the UE solution varies within the solution set. We show that such a risk-neutral scheme can be formulated as a stochastic program, which complements the traditional risk-prone SBTP approach and the risk-averse SBTP approach we developed recently. The proposed model can be solved by a simulation-based optimization algorithm that contains three major steps: characterization of the UE solution set, random sampling over the solution set, and a two-phase simulation optimization step. Numerical results illustrate that the proposed risk-neutral design scheme is less aggressive than the risk-prone scheme and less conservative than the risk-averse scheme, and may thus be more preferable from a toll designer's point of view.

1. Introduction

Congestion pricing charges motorists a toll for using a particular stretch of highway or for entering a particular area (e.g., "cordon tolls" for access to urban areas). It is to apply economic principles (i.e., pricing) to solve traffic congestion or related problems (Knight, 1924; Pigou, 1932; Vickrey, 1969; Yang and Huang, 2005). Congestion pricing is considered as one of the most promising approaches to address traffic congestion that has become not only an increasingly critical problem to our quality of life but also has serious consequences in terms of economic development (FHWA, 2007). In this article, we focus on link-based toll pricing, i.e., to determine optimal tolls of the entire set or a subset of network links to achieve certain system management objective.[1] Such problems can be broadly categorized as static pricing and dynamic pricing. This article focuses on the former, while the reader can refer to Friesz et al. (2007); Lu et al. (2008); Ban and Liu (2009) for recent advances on dynamic toll pricing. Static toll pricing can be further classified as first best toll pricing (FBTP) and second best toll pricing (SBTP). FBTP assumes that all network links can be tolled. Early works on FBTP aimed to determine optimal tolls by applying economic principles such as marginal cost (Beckmann et al., 1956; Arnott, 1979; Smith, 1979). Hearn and Ramana (1998) formulated FBTP as network equilibrium models and found that other optimal toll schemes exist, including the marginal cost toll as a special case.

This article focuses on SBTP, which assumes that tolls can only be imposed on a subset of network links due to technical, policy, or other constraints. It was noted in Johansson-Stenman and Sterner (1998) that SBTP is a trade-off between

* Corresponding author. Tel.: +1 5182768043; fax: +1 5182764833.
E-mail addresses: banx@rpi.edu (X. Ban), ferris@cs.wisc.edu (M.C. Ferris), lmtang@wisc.edu (L. Tang), shulu@email.unc.edu (S. Lu).

[1] Note that this does not necessarily mean that the resulting network model has to be link-based. In certain cases, path-based formulations have to be used; see the second example in Section 6.

0191-2615/$ - see front matter
http://dx.doi.org/10.1016/j.trb.2012.11.003

efficiency gains and system investment/operation costs. The SBTP with two-route problem, one tolled and one un-tolled, was studied in Marchand (1968), Verhoef et al. (1996), and Liu and McDonald (1999). Yang and Lam (1996) proposed a bi-level model to determine optimal tolls for a subset of network links by considering queuing effects. Since then, SBTP has been extensively studied (Verhoef, 2002a,b; Zhang and Ge, 2004; Larsson and Patriksson, 1998; Brotcorne et al., 2001). In particular, the elastic demand SBTP problems were investigated in Verhoef (2002a,b). Different design objectives such as those of the system manger and road users were explored in Zhang and Ge (2004) for variable (i.e., elastic) demand SBTP. SBTP with stochastic user equilibrium in the lower level was also studied in Sumalee et al. (2006). Furthermore, while most of existing SBTP models require both traffic demands and link costs as given (either fixed or as a given function), Yang et al. (2009) recently proposed a pricing scheme that can determine the optimal tolls for a set of links without knowing the exact demand or link cost information. For more comprehensive reviews on SBTP, readers may refer to Yang and Huang (2005) and Lawphongpanich et al. (2006).

Most existing SBTP models on general traffic networks are formulated as bilevel problems (Bard, 1998) or mathematical programs with equilibrium constraints (MPEC, see Luo et al. (1996)): the upper level is to optimize a system objective, while the lower level is to solve an user equilibrium (UE) problem to account for drivers' choice behaviors (such as route choice). Although there are significant variations among the existing SBTP models, in essence, SBTP can be formulated as the following general form:

$$\begin{aligned} min_{y,x} \quad & Q(y,x). \\ \text{s.t.} \quad & y \in K_y \\ & x \text{ solves } VI(V(y,x), K_x) \end{aligned} \tag{1}$$

Here $VI(V(y,x), K_x)$ denotes a variational inequality (VI) defined as "finding $x \in K_x$ such that $V^T(y,x)(x' - x) \geqslant 0, \forall x' \in K_x$." The two defining sets K_x and K_y, usually convex and compact, are for the *lower* level variable x and the *upper* level variable y respectively. In the SBTP setting, x can be considered as the vector of link flows or path flows, while y is the link toll vector. Notice that both the upper level system objective (i.e., $Q(y,x)$) and the defining function of the lower level VI (i.e., $V(y,x)$) are functions of (y,x). Also $Q(y,x)$ can take various forms besides the total system travel time depending on the system manager's objective (Zhang and Ge, 2004). For example, a weighted total system travel time is used in Ban et al. (2009) and Ban and Liu (2009) by assigning different weights on different link travel times. In the second example in Section 6 of this article, the objective is to minimize total system emissions.

Model (1) is an MPEC, for which theories and solution algorithms have been extensively studied in the mathematical programming literature. See Luo et al. (1996) for a review, and Ralph and Wright (2004) and Ferris (2004) for more recent advances on theories and solution techniques respectively. Loridan and Morgan (1988) observed that in case the lower level problem (formulated as a nonlinear program (NLP) in Loridan and Morgan (1988) instead of a VI) has multiple solutions for a given y, different lower level solutions may result in different upper level objective values. Model (1) in this case simply focuses on the VI solution that produces the minimum of all these objective values. Model (1) was then called the *optimistic* design scheme (Loridan and Morgan, 1988) as it aims to optimize for the best case scenario, i.e., the minimum of the objectives as VI solution varies within its solution set. Accordingly, a *pessimistic* design scheme was proposed (Loridan and Morgan, 1988) to optimize for the worst case scenario, i.e., the maximum of the objectives as VI solution varies. Ban et al. (2009) explicitly studied this issue for SBTP and observed that if the UE solution is nonunique, different UE solutions may result in different upper level objective values depending on the specific functional form of $Q(y,x)$. As a result, the UE solution set represents an uncertainty set *from the toll designer's perspective* when designing optimal tolls (Ban et al., 2009). In this sense, most existing SBTP models is risk-prone or optimistic if UE solution is nonunique (unless the upper level objective admits a unique value over the entire UE solution set). To address this issue, a risk-averse SBTP scheme was developed in Ban et al. (2009) to optimize for the maximum of the objective values as UE solution varies, similar to the pessimistic scheme in Loridan and Morgan (1988). The risk-averse scheme in Ban et al. (2009) however is based on a VI formulation in the lower level, which extends the scheme in Loridan and Morgan (1988) that is based on an NLP in the lower level.

The risk-averse SBTP scheme in Ban et al. (2009) is formulated as a robust optimization problem; similar schemes have also been applied recently to bounded rationality UE (BRUE) (Lou et al., 2010) and dynamic network design problems (Chung et al., 2011). It is well known that the results obtained by solving a robust optimization model is generally too conservative (or too pessimistic) (Yin and Lawphongpanich, 2007), while the risk-prone scheme is too aggressive (or too optimistic). This is because both schemes consider only extreme cases, i.e. a single point in the UE solution set. To address this issue, we propose in this article a "risk-neutral" scheme that explicitly considers the entire UE solution set when designing SBTP. The risk-neutral scheme aims to optimize for the expected objective value as the UE solution changes within the solution set. In particular, by associating certain probability distribution function to the realization of the UE solution over its solution set, we formulate the proposed risk-neutral model as a stochastic program, similar to the problem studied in Deng (2007) and Deng and Ferris (2009). In this setting, the UE solution set and the set of its subsets constitute the sample space. In Deng (2007) and Deng and Ferris (2009), the sample space is fixed. Our risk-neutral problem however has a changing sample space because the UE solution set varies with the toll vector. Hence, the risk-neutral model we study in this article extends the model and results in Deng (2007) and Deng and Ferris (2009).