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Global optimization methods for the discrete network design problem

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ABSTRACT

This paper addresses the discrete network design problem (DNDP) with multiple capacity levels, or multi-capacity DNDP for short, which determines the optimal number of lanes to add to each candidate link in a road network. We formulate the problem as a bi-level programming model, where the upper level aims to minimize the total travel time via adding new lanes to candidate links and the lower level is a traditional Wardrop user equilibrium (UE) problem. We propose two global optimization methods by taking advantage of the relationship between UE and system optimal (SO) traffic assignment principles. The first method, termed as SO-relaxation, exploits the property that an optimal network design solution under SO principle can be a good approximate solution under UE principle, and successively sorts the solutions in the order of increasing total travel time under SO principle. Optimality is guaranteed when the lower bound of the total travel time of the unexplored solutions under UE principle is not less than the total travel time of a known solution under UE principle. The second method, termed as UE-reduction, adds the objective function of the Beckmann-McGuire-Winsten transformation of UE traffic assignment to the constraints of the SO-relaxation formulation of the multi-capacity DNDP. This constraint is convex and strengthens the SO-relaxation formulation. We also develop a dynamic outer-approximation scheme to make use of the state-of-the-art mixed-integer linear programming solvers to solve the SO-relaxation formulation. Numerical experiments based on a two-link network and the Sioux-Falls network are conducted.

1. Introduction

Streets and highways in transportation networks are regularly improved and/or expanded to satisfy the rapidly growing travel demand and to alleviate the congestion problem. The network design problem (NDP) involves the optimal allocation of budget to the expansion of existing links and/or to the addition of new candidate links so as to optimize the network performance (e.g. minimizing the total travel time or generalized cost) while accounting for the travelers' route choice behavior which is in a user equilibrium (UE) or stochastic user equilibrium (SUE) manner (Yang and Bell, 1998, 2001). Depending on the nature of road capacity change, the NDP can be categorized into the continuous network design problem (CNDP), in which the capacity of links is modeled as continuous decision variables (see, e.g., Abdulaal and LeBlanc, 1979; Suwansirikul et al., 1987; Friesz et al., 1992; Meng et al., 2001; Chiou, 2005; Li et al., 2012; Wang et al., in press), the discrete network design problem (DNDP), in which the capacity of links can only take a set of given numbers, and the mixed

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discrete network design problem (MNDP), which is a combination of CNDP and DNDP (see Yang and Bell, 1998). The NDP can be formulated as a bi-level programming model, in which the upper level is to determine the network design to optimize network performance, and the lower level is usually the UE problem. Due to the intrinsic complexity, the NDP has been recognized as one of the most challenging problems in transportation. It is also a subject of a substantial stream of research over a few decades (see Magnanti and Wong (1984), Friesz (1985), and Yang and Bell (1998) for comprehensive surveys).

LeBlanc (1975) developed a bi-level programming formulation to describe the DNDP, in which, the upper-level is to minimize the total system cost subject to the investments on new links, and the lower-level is the UE problem with fixed demand. He presented a branch-and-bound (B&B) algorithm for solving the DNDP, in which the 0–1 branching decision variables indicate whether a potential link should be added to the network. An upper bound of the total travel time is obtained when an integer solution is obtained. In view of the Braess' Paradox that adding a link may actually increase the total travel time when travelers' route choice behavior follows the UE principle, LeBlanc (1975) derived a valid lower bound for a branch by requiring that travelers' route choice behavior follows the system optimal (SO) principle. The algorithm proposed by LeBlanc (1975) is relatively inefficient because the lower bounds are relatively loose. Farvaresh and Sepehri (2013) developed a revised B&B algorithm based on LeBlanc (1975). Poorzahedy and Turnquist (1982) approximated the objective function of the DNDP and transformed the bi-level model into a single one. A heuristic algorithm based on the B&B algorithm was given. In general, the difficulty in applying the B&B method lies in how to adapt the methodology to deal with the intrinsic non-convexity of the DNDP due to the implicit, nonlinear UE constraints.

Magnanti and Wong (1984) proposed a unifying framework for a number of algorithms. Within the framework, the Lagrangian relaxation and dual ascent procedures are effective in providing bounds for some instances of DNDP. Chen and Alfa (1991) investigated the DNDP using a logit-based stochastic incremental traffic assignment approach. Gao et al. (2005) proposed a solution algorithm by using the support function concept to transform the upper-level programming model to a usual nonlinear programming problem. Farvaresh and Sepehri (2012) formulated the UE constraint as the equivalent Karush–Kuhn–Tucker (KKT) condition. The KKT condition is then linearized by introducing binary decision variables.

Wang and Lo (2010) formulated the CNDP and DNDP as a single-level optimization problem with complementary constraints. This formulation is transformed to a mixed-integer nonlinear optimization model by enumerating the paths between origin–destination (OD) pairs. The resulting mixed-integer nonlinear model is further linearized through piecewise linear approximation of link travel time functions with binary variables. Luathep et al. (2011) formulated the MNDP as a single-level optimization problem with a variational inequality (VI) constraint representing the UE condition. The VI constraint, which needs to be satisfied for all feasible link flows, is proved to be sufficient if it is satisfied for all the extreme points of the set of feasible link flows. The large number of extreme points can be generated dynamically.

There are also heuristic approaches for the DNDP in the literature, for example, genetic algorithm (Drezner and Wesolowsky, 2003; Cantarella and Vitetta, 2006; Jeon et al., 2006), ant system method (Poorzahedy and Abulghasemi, 2005), and hybrid meta-heuristics (Poorzahedy and Rouhani, 2007; Miandoabchi et al., 2012).

The considered NDP belongs to a family of difficult bi-level programming problems in the mathematical programming literature, which is usually tackled through reformulation into a single-level problem (reader may refer to Dempe (2003) and Colson et al. (2005) for review on bi-level programming problems). One approach is to replace the lower-level decisions by an implicitly determined function (reaction function), and another possible approach is to replace the lower-level problem by its KKT conditions. The resulting equivalent single-level problem is solved by methods such as sequential quadratic programming, descent methods, branch-and-bound, penalty function methods, and trust-region methods. However, in many cases only a local optimum can be guaranteed. Moreover, most research works on bi-level programming so far have overwhelmingly focused on problems with only continuous variables in both the upper and lower level. When discrete decision variables exist in either the upper or the lower level problem, the bi-level model would be even more difficult yet challenging, which is the case for the DNDP. Therefore, more efficient methods are deemed necessary, particularly in view of the fact that only very limited attention has been paid so far to the DNDP.

In this paper, we consider the DNDP with multiple capacity levels, or multi-capacity DNDP for short. The transport authority needs to determine how many lanes should be added to each link from a set of candidate links. The decision variable, that is the number of lanes, takes discrete values, and therefore this problem falls into the category of DNDP. The objective of this study is to develop efficient global optimization methods for the multi-capacity DNDP. As will be shown later, the multi-capacity DNDP nests the conventional DNDP as a special case. This paper first relaxes the bi-level programming model by formulating a single-level problem using the SO-relaxation. Two novel and efficient global optimization methods based on the relationship between UE and SO principles are developed. The first method, termed as SO-relaxation, takes advantage of the property that an optimal network design decision with SO traffic assignment can be regarded as a good approximate solution with UE traffic assignment. The second method, termed as UE-reduction, reduces the gap between the bi-level programming model and the single-level model by adding valid inequalities based on the objective function of the Beckmann-McGuire-Winsten transformation of the UE traffic assignment problem (Beckmann et al.,1956). The rest of the paper is organized as follows. The bi-level programming formulation is presented in Section 2. Section 3 presents the two global optimization methods. In Section 4, two numerical examples are reported. Section 5 concludes the paper.