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On a mean field game approach modeling congestion and aversion in pedestrian crowds

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ABSTRACT

In this paper we present a new class of pedestrian crowd models based on the mean field games theory introduced by Lasry and Lions in 2006. This macroscopic approach is based on a microscopic model, that considers smart pedestrians who rationally interact and anticipate the future. This leads to a forward-backward structure in time. We focus on two-population interactions and validate the modeling with simple examples. Two complementary classes of problems are addressed, namely the case of crowd aversion and the one of congestion. In both cases we describe the model and present numerical solvers (based on the optimization formulation and the partial differential equations respectively). Finally we present numerical tests involving anticipation phenomena and complex group behaviors such as lane formation.

1. Introduction

In the last decades crowd motion modeling has become an active area of research. As the world population and the urbanization continues to increase we cannot underestimate the importance of understanding the behavior of human crowds. Nowadays there is lot of interdisciplinary research among physicists, sociologists, biologists and mathematicians; crowd motion has become one of the emerging research topics.

In this paper we present a new approach to model crowd motion. It is called mean field games (MFG) and has been introduced initially by Lasry and Lions, see Lasry and Lions (2006a,b, 2007). A particular appealing feature is that the approximating macroscopic mean field model is derived from a mean-field game as the number of players tends to infinity. Consequently it offers the tractability of a macroscopic framework together with the more realistic interpretation at the microscopic level. Furthermore we treat players as "smart" individuals which try to optimize their path with respect to a particular goal. Thus, a MFG model is quite novel for crowd modeling and differentiates itself from other models mainly in the combination of two points. First we treat the pedestrians as real individuals or agents, having preferences (they are utility maximizer) and strategical interactions within the crowd. They do not obey certain predefined laws but rather choose their actions (velocity and directions) after evaluating the general situation. Note that we use the term agents as well as other terminology from economics. Secondly the individuals anticipate the future. This is mathematically expressed through a forward-backward structure, the forward dynamic describing the crowd dynamic whereas the backward one is needed to build the expectations. This differs from pure forward frameworks that might be unable to describe particular features of crowd dynamics. In economics, this is related to the well-known and widely accepted notion of *rational expectations*

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hypothesis, which basically refers to the validity of approximative predictions in markets with many agents. It has been introduced by Muth (1961) and deeply studied by Lucas et al., see Lucas and Sargent (1981). In the proposed crowd motion model, the rational expectations assumption is not perfectly correct since it assumes that any agent knows and anticipates the distribution of other agents. But making this assumption is a necessary first step before providing more realistic models. For a more comprehensive discussion on rational expectation equilibria in the present work, we refer to Section 3.

We would like to mention similar approaches which have been introduced in the literature (see Section 2 for an elaborate overview on crowd motion models). Hoogendoorn (2002), Hoogendoorn and Bovy (2003) used a differential game and utility maximization approach in a finite and deterministic setting. Nevertheless, the authors do not consider mean-field type interactions with rational expectations. The idea of using MFG as a tool to model crowd dynamics has been brought up in very recent works, see Guéant (2009), Guéant et al. (2010), Lachapelle (2010), Dogbé (2010). The novelty of this paper is threefold: we consider two-group interactions, we address congestion cases and finally we try to settle the MFG approach in the landscape of some existing models. We believe that a MFG model is interesting in the sense that beyond offering a description of how pedestrian behave in a crowd, it also answers the questions why and how the crowd moves (or why it has its specific shape in the stationary case). This is given by the feedback strategy, resulting from the backward reasoning. As indicated in the name mean field games are a game approach and it has the very important feature to consider the pedestrians' behavior as a non-cooperative equilibrium. Therefore it is unfortunately not possible to consider cases of small groups of people interacting within the crowd.

The presented approach is based on an N-player mean field game. The limiting macroscopic partial differential equations (PDEs) verified by a mean field equilibrium have a simple structure, which allows the development of efficient numerical methods. This is probably one of the main advantages of the approach compared to many agents models: MFG can be solved with much lower computational cost than games with a finite number of agents. Different numerical approaches for MFG can be found in the literature (see for instance Achdou and Capuzzo-Dolcetta, 2010; Guéant, 2009; Lachapelle, 2010; Lachapelle et al., 2010; Lions, 2008–2010). They are either based on the optimal control formulation of the problem or on the PDE system. In the first case different methods from parabolic optimal control theory are available. On the PDE level Newton-type methods have been used successfully for numerical simulations. Nevertheless the forward-backward structure and the highly nonlinear equations still pose a great challenge to numerical analysts and the development of efficient numerical methods is still in its early stages.

This paper is organized as follows. We first provide in Section 2 a brief review of some important models used for crowd motion modeling. Section 3 is devoted to the general introduction of mean field games in a crowd dynamic framework. Next we turn to the central topic of our paper, namely the study of two interacting populations (or equivalently crowds). More particularly we present two different classes of MFG (in order to show the diversity of cases that could be treated by MFG). For both of them we describe the modeling, discuss the numerical solving strategies and provide numerical results. In Section 4 we present a model where individuals are averse to the crowd, in a time-dependent framework. The gradient-descent numerical method is based on an optimal control formulation of the problem. Our numerical tests mainly provide a symmetry breaking in some xenophobic situation (where the pedestrians of the first group avoid the pedestrians of the other group). In Section 5 we look at a two-species stationary model involving congestion. Here the numerical simulations, based on Newtons' method, reproduce well known phenomena in crowd motion like lane formation. We also provide an example involving anticipations, that is a situation where the forward-backward setting is crucial. Finally, we close with some concluding remarks in Section 6.

2. Overview on mathematical models for pedestrian flow

In this section we give an overview on the literature of crowd motion models to depict the scientific framework the presented paper fits in. The first empirical studies on crowd motion have been deducted about 50 years ago (cf. Hankin and Wright, 1958; Hoel, 1968), nowadays various modeling approaches can be found in the literature. We distinguish between four different modeling approaches, i.e. *microscopic*, *mesoscopic*, *macroscopic* and *multi scale* models.

Microscopic models treat every pedestrian as a single agent, whose motion is determined by reaching a desired destination and the interaction with the surrounding agents. One of the most popular models in this framework is the behavioral force model (cf. Helbing and Molnar, 1995; Helbing et al., 2002; Helbing and Johansson, 2009). Here the motion of the *i*th agent is determined by Newtons' laws of motion:

$$m_i \dot{v}_i(t) = F_i(t), \tag{1}$$

where v_i denotes his\her velocity and F_i the force acting on him\her. The force field F_i includes the preferred direction of motion and the level of comfort or discomfort at certain distances to other pedestrians. Behavioral force models have been very successful in describing various crowd motion phenomena like lane formation, oscillations at bottlenecks or clogging, but do not include active self-thinking or involvement of agents.

Hoogendoorn and Bovy proposed a game theoretic (with optimal control) approach, where every agent tries to maximize a subjective utility function, see Hoogendoorn and Bovy (2003). Here every agent tries to predict the behavior of the surrounding agents in the near future and continuously reconsiders her walking choice using current observations. The main difference with our work is that it is only a microscopic approach where the authors do not consider mean field interactions.