Transportation Research Part B 45 (2011) 1641–1659

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Multi-class percentile user equilibrium with flow-dependent stochasticity

Yu (Marco) Nie *

Department of Civil and Environmental Engineering, Northwestern University, 2145 Sheridan Road, Evanston, IL 60208, United States

ARTICLE INFO

Article history:
Received 19 October 2010
Received in revised form 5 June 2011
Accepted 5 June 2011

Keywords:
Percentile user equilibrium
Traffic assignment
Variational inequality
Convolution
Flow-dependent stochasticity

ABSTRACT

Travelers often reserve a buffer time for trips sensitive to arrival time in order to hedge against the uncertainties in a transportation system. To model the effects of such behavior, travelers are assumed to choose routes to minimize the percentile travel time, i.e. the travel time budget that ensures their preferred probability of on-time arrival; in doing so, they drive the system to a percentile user equilibrium (UE), which can be viewed as an extension of the classic Wardrop equilibrium. The stochasticity in the supply of transportation are incorporated by modeling the service flow rate of each road segment as a random variable. Such stochasticity is flow-dependent in the sense that the probability density functions of these random variables, from which the distribution of link travel time are constructed, are specified endogenously with flow-dependent parameters. The percentile route travel time, obtained by directly convolving the link travel time distributions in this paper, is not available in closed form in general and has to be numerically evaluated. To reveal their structural properties, percentile UE solutions are examined in special cases and verified with numerical results. For the general multi-class percentile UE traffic assignment problem, a variational inequality formulation is given and solved using a route-based algorithm. The algorithm makes use of the diagonal elements in the Jacobian of percentile route travel time, which is approximated through recursive convolution. Preliminary numerical experiments indicate that the algorithm is able to achieve highly precise equilibrium solutions.

1. Introduction

Urban transportation systems are affected by uncertainties of various sorts, which can be broadly classified as those affecting the supply of transportation (e.g. weather, accidents, natural and man-made disasters) and those associated with the demand for transportation (e.g. travel and activity behavior, special events). Taken individually or in combination, these factors could adversely affect the quality of transportation services. Travel behavior researchers have established that unanticipated long delays on highways typically produce much worse frustration among motorists than “predictable” ones. To hedge against travel time fluctuations, travelers usually budget a sizable buffer time. In 1982, a 20-min free-flow trip in the US requires on average an extra 12-min buffer time if on-time arrival is important (FHWA, 2005). A similar trip would require 60% more buffer time in 2003. An important question that often arises in the context of transportation planning and network design is: how travelers’ “buffer-reserving” behavior to cope with network uncertainties affects the overall performance of the transportation system. This paper provides a basis to address this question by formulating and solving a traffic assignment model that incorporates such behavior.

Traffic assignment is widely used to generate facility-level predictions by mapping travel demands onto the transportation network. The classic model, which postulates that travelers always choose the shortest route (Wardrop, 1952), produces a steady network flow pattern known as *user equilibrium* (UE) as the solution to the traffic assignment problem (Beckmann et al., 1956; Sheffi, 1985). Despite its remarkable success in practice, the classic traffic assignment model ignores the

* Tel.: +1 847 467 0502; fax: +1 847 491 4011.
E-mail address: y-nie@northwestern.edu

0191-2615/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.trb.2011.06.001

uncertainties inherited in both supply and demand sides of the transportation, which has been frequently cited as one of its major limitations. Many models attempt to incorporate uncertainties in traffic assignment. This paper is focused on the impacts of supply uncertainties on travelers' route choices, that is, the only source of uncertainties considered herein is from the supply of transportation. More specifically, we assume that the service flow rate (SFR)[1] of each road segment is random, and subject to the influence of exogenous (e.g. weather) and endogenous (e.g. traffic breakdown) random factors. Fluctuations in travel demands as well as individuals' imperfect information and irrational choice behavior are excluded to simplify the analysis and maintains the tractability of the resulting model.

In this paper, the travel time on each link of the transportation network is treated as an *independent* random variable, whose distribution is endogenously determined from the distribution of SFR and a functional relationship between SFR, traffic flow and travel time. Accordingly, the route travel time is also a random variable, and its distribution is formulated by convolving the distributions of its member links. Travelers are assumed to have perfect knowledge of the route travel time distributions, and choose best routes to fulfill their desired on-time arrival probability, i.e., the probability of arriving at the destination on-time or earlier. Clearly, the "best" route in this context is the route that requires least percentile travel time (or time budget) corresponding to the desired on-time arrival probability. Consequently, the Wardrop equilibrium can be extended to the *percentile user equilibrium*, where all travelers with the same on-time probability should experience minimum and equal percentile travel time between the same origin–destination (O–D) pair.

Although the percentile user equilibrium is not a new concept, several features distinguish this study from those found in the literature.

- First of all, the proposed model enables flow-dependent stochasticity, which means that the distribution of SFR is endogenously determined based on link flow. In contrast, previous studies usually require such distributions to be specified exogenously, independent of traffic flow. As explained in detail in Section 6, the distribution of random road capacities is likely to be flow-dependent because the two important contributing factors of stochasticity, accident rates and traffic breakdown, are known to be affected by traffic flow.
- Second, the percentile route travel time is modeled as an implicit function of the route flow, which is evaluated by adding distributions directly through convolution. We postulate that the implicit percentile route travel time function is not monotone in general and provide supporting numerical evidence. A sufficient but restrictive condition that ensures monotonicity of the function is provided. Most previous studies depend on the central limit theorem (CLT) to evaluate this function. For one thing, CLT requires imposing restrictions on the type of distributions that can be used.[2] Moreover, since the solution provided by CLT is exact only when the number of distributions n approaches infinity, its approximation errors may be unsatisfactorily large when n is small.
- Thirdly, analytical solutions are examined in special cases in order to provide insights to understanding the properties of the general percentile UE solutions. These solutions are then verified through numerical experiments.
- Finally, a route-based algorithm is proposed for the percentile UE problem. The algorithm makes use of the partial Jacobian of the percentile route travel time, which is approximated by convolving the distributions of the link travel time derivatives. While not implemented and tested in this paper, the proposed algorithm can be combined with a column generation scheme to obviate path enumeration. One possibility is using the reliable a priori shortest path algorithm (Nie and Wu, 2009) to find all non-dominated paths, which collectively provide minimum percentile travel time for any on-time arrival probability.

The remaining of this article is organized as follows. Section 2 briefly reviews the related studies. Section 3 discusses the adopted stochastic performance model and its analytical properties. Section 4 characterizes the percentile UE and presents an equivalent formulation based on variational inequality (VI). The properties of the percentile route travel time function and the evaluation of its Jacobian are also presented in Section 4. Section 5 reveals the structure of the percentile UE solutions in special cases. Section 6 discusses flow-dependent stochasticity. The solution algorithm and numerical results are presented in Sections 7 and 8, respectively. Section 9 concludes the study with remarks on future research.

2. Literature review

2.1. Classic stochastic user equilibrium models

It is well known that travelers do not always choose the best route, owing to random disturbances such as measurement and perception errors. Probabilistic choice theory most often used in transportation postulates that (1) travelers always want to choose the best route and (2) each route has a probability of being the best. Accordingly, a traffic assignment problem can be formulated to achieve *stochastic user equilibrium* (SUE) (Daganzo and Sheffi, 1977; Fisk, 1980), where the proportion of

[1] We use service flow rate instead of capacity because random disruptions, such as accidents and traffic breakdown, affect service flow rate but not capacity. Notes in Acknowledgement explains why SFR might be more precise term than "capacity" when used in the circumstances such as concerned in this paper.

[2] In the most restrictive form, CLT requires that all distributions are independently and identically distributed (IID). When the distributions are not identically distributed, other conditions, such the Lyapunov condition or the Lindeberg condition, are needed to ensure the validity of the theorem (Ash and Dolans-Dade, 2000).