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Complementarity formulations for the cell transmission model based dynamic user equilibrium with departure time choice, elastic demand and user heterogeneity

Lanshan Han, Satish Ukkusuri *, Kien Doan

School of Civil Engineering, Purdue University, West Lafayette, IN 47907–2051, USA

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A B S T R A C T

In this paper we formulate the dynamic user equilibrium problem with an embedded cell transmission model on a network with a single OD pair, multiple parallel paths, multiple user classes with elastic demand. The formulation is based on ideas from complementarity theory. The travel time is estimated based on two methods which have different transportation applications: (1) maximum travel time and (2) average travel time. These travel time functions result in linear and non-linear complementarity formulations respectively. Solution existence and the properties of the formulations are rigorously analyzed. Extensive computational experiments are conducted to demonstrate the benefits of the proposed formulations on various test networks.

1. Introduction and literature review

As a mathematical paradigm for transportation network analysis on congested networks, Dynamic Traffic Assignment (DTA) has enjoyed a significant research activity and broad consensus over the last few years. A comprehensive review of various Dynamic Traffic Assignment models can be found in Peeta (1994), Peeta and Ziliaskopoulos (2001). Over the last few years, various approaches have been proposed to formulate and solve the DTA problem. Typically, these problems are classified under two approaches: mathematical programming and simulation. The mathematical programming approach focuses on developing rigorous analytical formulations and the characterization of equilibrium therein while the simulation approaches focus on traffic realism and equilibrium computation often times at the expense of mathematical rigorousness. In addition, DTA models differ in the specific assumptions of the underlying models related to the modeling of time steps (discrete or continuous time) and the flow variables (discrete or continuous flow). Typically most formulations in the literature use continuous flow with discrete or continuous time network models.

In the literature, DTA models have been developed using mathematical programming techniques (e.g., Friesz et al. (1989), Merchant and Nemhauser (1976)). This approach allows foundational analysis and mathematical characterization of properties such as existence and uniqueness and clearly guarantee conditions such as the first-in-first-out (FIFO) which are important for tractability. Depending on the specification of the objectives, these models can be used for both descriptive and prescriptive purposes. One key limitation of the analytical formulations is the simplified representation of traffic

* Corresponding author.
E-mail addresses: han144@purdue.edu, lanshanhan@gmail.com (L. Han), sukkusur@purdue.edu (S. Ukkusuri), kdoan@purdue.edu (K. Doan).

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dynamics in the network loading procedure. In the literature, typically three network loading procedures are primarily studied: (1) link exit functions akin to the Bureau of Public Road (BPR) link performance function for static traffic assignment. The use of this network loading procedure can be found in Ban et al. (2008), Carey (1987, 1992), Friesz et al. (1989), Merchant and Nemhauser (1976), Ran et al. (1993), Wu et al. (1998), (2) point queue model, which is primarily used in the single bottleneck models Arnott et al. (1993, 1994), Hendrickson and Kocur (1981), Newell (1987), Ramadurai (2009), Ramadurai et al. (2010), Verhoef and Small (2004), Vickrey (1969), and (3) physical queue model, which is mainly based on the spatial queue models such as the cell transmission model (Ramadurai, 2009; Ramadurai et al., 2009; Ukkusuri and Waller, 2008; Ziliaskopoulos, 2000). The use of these different traffic models lead to models with varying traffic realism, the spatial queuing models being the most realistic. However as noted by some authors (Ukkusuri and Waller, 2008) this makes the already complex DTA model highly intractable.

Since the original work of Merchant and Nemhauser (1976), there have been many advancements in the study of DTA. Friesz et al. (1989) studied the dynamic user equilibrium and dynamic system optimal formulation using complementarity theory and the network loading is based on the link exit flow functions. These models played an important role in the development of DTA models. However, as noted by Carey (1987, 1992), Carey and Subrahmanian (2000), Wu et al. (1998), in certain instances, particularly when a small flow is followed by a large flow, the link exit model violate the first-in first-out (FIFO) property, which is important for any reasonable traffic flow model. In addition, this model lacks solution procedure for the general network (Peeta, 1994; Peeta and Ziliaskopoulos, 2001). Further, as pointed out by Daganzo (1995), a potential shortcoming of DTA models is the lack of traffic realism in network loading procedures with their inability to capture link spillover and shockwave propogation. Other studies (Lo and Szeto, 2002a,b) confirm the importance of using realistic traffic flow models within the DTA model.

In the last decade, the cell transmission model (CTM) by Daganzo (1994, 1995) has been used within DTA to capture shockwave and link propagation properties. It is a mesoscopic traffic model in which the physical length of roadways is divided into a number of cells. It takes into account the traffic properties such as flow and density and thereby captures link spillovers and shockwave propagation. Ziliaskopoulos (2000) used the CTM to develop a system optimal problem as a linear programming. Ukkusuri and Waller (2008) compared the dynamic user equilibrium and dynamic system optimal by linear formulation with different objective functions. These formulations however approximate the non-linear propagation of flow with linear constraints leading to the "holding back" problem. Lo and Szeto (2002a,b), Szeto and Lo (2004) embedded CTM model to solve the DTA problem.

Departure time choice is another dimension which is not fully captured within most DTA models. A well known framework for modeling the departure time is the single bottleneck (SB) model originally proposed by Vickrey (1969). Arnott et al. (1993, 1994) considered the continuous time formulation for homogeneous commuter SB model with elastic demand. Recently, Ramadurai et al. (2010) proposed the discrete time formulation to solve the dynamic user equilibrium of the single bottleneck model with heterogeneous commuters. The existence and uniqueness of the solution are proved. A potential drawback of these models is that they use point queue models for traffic flow. However, as observed by the authors (Ramadurai et al., 2010), there is no difference in the solutions of the SB model with a point queue or a spatial queueing model such as CTM since there is no spillback. However, there will be a significant difference in general networks.

This study develops a cell-based dynamic user equilibrium for a single origin destination (OD) and considers departure time choice, user heterogeneity and demand elasticity. The formulation is based on new techniques from complementarity theory (Cottle et al., 1992; Facchinei and Pang, 2003). This novel formulation offers a formal framework for the rigorous study and solution of dynamic equilibrium problems with a fully integrated cell transmission model, enabling the adoption of well established complementarity theory and methods to analyze and solve the model. The development of such a formulation will allow the clear characterization of the existence of dynamic equilibrium in single OD networks. Our eventual goal is to extend this mathematical framework to a general networks with multiple OD pairs and departure time choice. The primary contributions of this work are: (1) to develop a complementarity formulation for the CTM based dynamic user equilibrium model with departure time choice, user heterogeneity and demand elasticity for the networks of single OD pair and multiple parallel paths; (2) to show the existence of the solution by complementarity theory and (3) to compute the numerical solution under various conditions of traffic demand and user group parameters. To the best of our knowledge, this is one of the first formulations which fully integrated the network-wide cell transmission model into the DUE framework in addition to departure time choice, user heterogeneity and demand elasticity. The properties of those features can be obtained based upon the results of the proposed model such as the relation between time choice, path choice and users' characteristic. There are several differences between the proposed model and related work in the literature on CTM based DTA, in particular the work by Lo and Szeto (2002a,b), Szeto and Lo (2004). First, even though Lo and Szeto (2002a,b), Szeto and Lo (2004) use the CTM in the complementarity problem (CP) or variational inequality (VI) based DUE model, their formulation does not explicitly capture CTM using complementarity constraints. Rather, they simulate the problem using the CTM model based on the CP or VI formulations. For example, in the cited studies, the travel time was considered as an explicit function of the departure rate. Ideally, when CTM is embedded, one should consider the travel times as a function of the cell occupancy variables which in turn are functions of the departure rate. In our study, the CTM is explicitly formulated fully within complementarity framework. Second, the demand elasticity considered in Szeto and Lo (2004) assumes the inverse demand function to be strictly decreasing. This assumption excludes the fixed demand case. In our study, such a restrictive assumption is not needed. Third, and most importantly, this study is build on rigorous mathematical analysis under minimal assumptions. For example, in Szeto and Lo (2004), it is assumed that the travel times are continuous in the departure rates.