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A computationally efficient methodology to characterize travel time reliability using the fast Fourier transform

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ABSTRACT

In this paper we present a novel methodology to assess travel time reliability in a transportation network, when the source of uncertainty is given by random road capacities. Specifically, we present a method based on the theory of Fourier transforms to numerically approximate the probability density function of the system-wide travel time. Except for noted pathological cases, any common continuous or discrete probability distribution can be used to model capacity uncertainty. Theoretical bounds on the approximation errors are formally derived, both for general distributions as well as for the specific instance of normally distributed capacities. These bounds provide valuable insights into the structure of the approximation errors and suggest ways to reduce them. From a practical point of view, we propose a procedure based on successively refining the computational grid in order to guarantee accurate approximations. The proposed methodology takes advantage of the established computational efficiency of the fast Fourier transform. In a numerical case study, we demonstrate that the results of the methodology are consistent with intuition.

1. Introduction

It is widely accepted that the reliability of a transportation system is critical for society at large (Berdica, 2002). In terms of transportation networks, the reliability literature has its roots in the study of what has become known as **connectivity or terminal reliability** (Wakabayashi and Iida, 1992; Bell and Iida, 1997; Asakura et al., 2003). This type of study examines the probability that specific Origin–Destination (OD) pairs in a network remain connected when links are subject to complete failures. Because of the binary character of the link performance (they are either in service or not, or more generally, provide an acceptable level of service or not), connectivity reliability tends to be more appropriate for extreme events (e.g. earthquakes). Moreover, these models tend to ignore congestion effects which are particularly relevant from a transportation planning perspective.

Travel time reliability relates to the probability that travel times remain below acceptable levels. The earliest studies in this area used extensive computer simulation to determine the reliability of travel times (e.g. Asakura and Kashiwadani, 1991), while later studies employed sensitivity analysis to reduce the computational burden (Du and Nicholson, 1997; Bell et al., 1999). In an attempt to further improve on the computational efficiency, Sumalee and Watling (2003) proposed an approach to obtain bounds on the reliability by only considering a subset of all possible scenarios. However, as the authors have noted, the approximation scheme is efficient only in situations where a large fraction of the probability mass is concentrated on a relatively small number of scenarios. Recently, the same authors proposed a novel approach based on sample

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space partitioning to obtain bounds on the reliability of travel time (Sumalee and Watling, 2008). Ng et al. (in preparation-a) also examined bounds on the reliability of travel time. A unique feature of their approach is that it is distribution-free in the sense that exact probability distributions (which might not be available in practice) are not needed in order to characterize the uncertainty. Their methodology only requires moment information and a set of finite intervals in which the uncertain quantities are hypothesized to reside. Due to its semi-analytical character, the methodology has been shown to be extremely computationally efficient. The output of reliability studies typically yields a single scalar performance index (e.g. the probability that the system travel time exceeds a *predetermined* threshold) as a summary of the overall system performance. Clark and Watling (2005) depart from this philosophy and constructed the entire Probability Density Function (PDF) of the system-wide travel time. With the entire PDF it becomes possible to evaluate the probability that the travel time exceeds *any* given value. Bell (2000) and Bell and Cassir (2002) also assumed a novel approach by taking a more pessimistic view in the assessment of travel time reliability. They proposed models based on game theory to assess the worst case performance of a network in terms of its travel time.

Chen et al. (1999) introduced the notion of **capacity reliability**, which is defined as the probability that the transportation system can accommodate a given demand level at an acceptable level of service. A comprehensive simulation-based framework to assess this particular form of reliability was presented and discussed in Chen et al. (2002). More recently, Sumalee and Kurauchi (2006) used the concept of capacity reliability to evaluate network reliability in the wake of a major disaster.

If there were no uncertainty in a transportation network, the above probabilistic reliability measures would either be zero or one. In the current literature, the sources of uncertainty in a transportation network are often categorized as **demand uncertainty** (e.g. Asakura and Kashiwadani, 1991; Waller et al., 2001; Clark and Watling, 2005; Sumalee and Kurauchi, 2006; Lam et al., 2008) or **capacity uncertainty** (e.g. Wakabayashi and Iida, 1992; Chen et al., 2002; Sumalee and Watling, 2003; Sumalee and Watling, 2008). A major theme in these studies is the assumption of statistical independence. Unless the study is simulation-based (Chen et al. (2002) assumed that capacity degradations were correlated continuous random variables; for a comprehensive review of this correlation-based approach, see Ng et al., in preparation-b), it is often necessary to make the independence assumption for mathematical tractability. However, as pointed out in Lo and Tung (2003), while it is true that for certain situations the modeling of dependencies is crucial for the validity of the reliability assessment (e.g. during floods and earthquakes when certain areas are simultaneously affected), for other situations, the assumption of independence might be justifiable (e.g. traffic accidents, parking violations). Apart from this, the validity of the independence assumption might also depend on the geographical region in which the network is located as certain regions are more prone to incidents that would give rise to dependent capacity reductions in a road network. Because of the challenging nature, only a very limited number of studies have attempted to relax the assumption of independence. For instance, by postulating a multivariate normal distribution for the link flows in a network, Clark and Watling (2005) were able to model dependencies in link flows. Sumalee and Watling (2003, 2008) used a cause-based failure framework to introduce correlations in the capacity degradations.

A final criterion to characterize the transportation network reliability assessment literature is travel behavior under uncertainty. Three assumptions can be found in the literature. The first possibility assumes that travelers exhibit **non-adaptive** behavior, i.e. they do not change their paths as a function of the unfolding scenario. One motivation for such behavior is that travelers simply do not have enough time to react because of the unpredictability of events (Clark and Watling, 2005). On the other extreme, Lo and Tung (2003) suggested that people have learned about the possible scenarios, based on which they have settled in a single, fixed, long-term equilibrium pattern accounting for the uncertainties. An immediate consequence of the non-adaptivity assumption is that only a single, representative run of traffic assignment (e.g. based on some nominal levels of demand and capacity) is needed to predict travel patterns. Another possibility hypothesizes that only the travelers on affected routes (i.e. routes whose capacities have reduced) have the ability to change their paths while travelling. The behavior underlying this type of **partial adaptivity** has been coined partial user equilibrium (Sumalee and Watling, 2003). The third and final possibility allows for **fully adaptive** behavior, i.e. for every single scenario travelers decide on a new path to follow (e.g. Chen et al., 2002). Clearly, there is still little consensus on travel behavior under uncertainty. Various network equilibrium models – with different behavioral assumptions – have been proposed in the literature to model this behavior (Yin and Ieda, 2001; Watling, 2002; Lo and Tung, 2003; Yin et al., 2004; Lo et al., 2006; Shao et al., 2006; Siu and Lo, 2008; Szeto et al., 2006; Zhou and Chen, 2008; Lam et al., 2008; Ng and Waller, in preparation).

Based on the above characteristics of reliability studies, we can categorize this paper as a study on **travel time reliability** induced by **independent capacity variations** under **non-adaptive behavior**. This work can be seen as a complement of some of the results presented in Lo and Tung (2003) and Lo et al. (2006) in which the Central Limit Theorem (CLT) was invoked to guarantee normality of the travel time (by assuming strictly positive capacities and the exclusion of pathological cases). However, in the more general case, it is not always feasible to rely on the CLT. In this paper we derive the PDF of the system-wide travel time without relying on the CLT (Section 6 provides examples in which the CLT fails to hold, whereas the proposed methodology remains valid), which leads us to a different positioning of the current work: a complement to Clark and Watling (2005) who obtained the PDF of the travel time under stochastic demand. Apart from the source of uncertainty, the proposed methodology is tremendously different. Clark and Watling assumed the multivariate normal distribution based on which they were able to obtain analytical expressions for the moments of the random travel time. The moments were subsequently used to obtain the PDF of the travel time by fitting a parametric family of PDFs known as Jonhson curves (Johnson, 1949). The underlying principles of our approach are based on the theory of Fourier transforms. Furthermore, we obtain a numerical approximation of the PDF rather than an analytical expression. Finally, while we