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Robust congestion pricing under boundedly rational user equilibrium

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ABSTRACT

This paper investigates congestion pricing strategies in static networks with boundedly rational route choice behavior. Under such behavior, users do not necessarily choose a shortest or cheapest route when doing so does not reduce their travel times by a significant amount. A general path-based definition and a more restrictive link-based representation of boundedly rational user equilibrium (BRUE) are presented. The set of BRUE flow distributions is generally non-convex and non-empty. The problems of finding best- and worst-case BRUE flow distributions are formulated and solved as mathematical programs with complementarity constraints. Because alternative tolled BRUE flow distributions exist, our congestion pricing models seek a toll vector or pattern that minimizes the system travel time of the worst-case tolled BRUE flow distribution. As formulated, the models are generalized semi-infinite min–max problems and we propose a heuristic algorithm based on penalization and a cutting-plane scheme to solve them. Numerical examples are presented to illustrate key concepts and results.

1. Introduction

Congestion pricing has been advocated as an efficient method for mitigating congestion since the seminal work by Pigou (1920) and Knight (1924) (see Lindsey, 2006 for a recent review). However, congestion pricing models in the literature (e.g., Yang and Huang, 2005) typically assume that users are perfectly or unboundedly rational, i.e., they always choose shortest (or cheapest) routes possible. However, when doing so only leads to a small or negligible improvement in their travel times, some users may not be sufficiently encouraged to change routes in practice. Behaving in this manner, users are said to be "boundedly rational." The literature in psychology and economics has provided a wide range of evidence that bounded rationality is important in many contexts, particularly in the context of day-to-day choices (see Conlisk, 1996 for a recent review on this topic).

Although there are many models with the bounded rationality assumption in the economic literature (see, e.g., Conlisk, 1996, and references cited therein), the number of such models in transportation is miniscule. Mahmassani and Chang (1987) study the travel and trip timing decisions by boundedly rational travelers in an idealized setting that consists of one origin–destination (O–D) pair and a single route. Later, Jayakrishnan et al. (1994), Hu and Mahmassani (1997), Mahmassani and Liu (1997), and Mahmassani (2000) apply the concept and similar models to study, e.g., the effects of advanced traffic information and management systems on road systems. Similar to Conlisk (1996), Nakayama et al. (2001) conclude that their experimental study indicates a need to evaluate the validity of the perfectly rational assumption in the traffic equilibrium analysis. Although referred to as "tolerance-based," Szeto and Lo (2006) are "forced" to consider bounded rationality in their dynamic traffic assignment problem because the problem may be infeasible when travelers are perfectly rational. In

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their paper, Szeto and Lo compare and contrast queuing paradigms in a dynamic setting, a setting under which many problems in transportation can be massively large, cannot be solved analytically, and often have to rely on computer simulation for solutions.

Instead of trying to incorporate more realism into, e.g., the description of the day-to-day or within-day dynamics, this paper addresses boundedly rational route choice behaviors in static networks where boundedly rational user equilibrium (BRUE) arises whenever all users' travel costs are not sufficiently larger than the best available ones and no user has an incentive to switch his or her route. The focus of this paper is to study the implication of BRUE in transportation systems planning and develop an analytical framework to proactively account for boundedly rational travel behaviors in the context of congestion pricing. We also note that there are other network equilibrium models in the literature that attempt to describe more realistically users' route choice behaviors, such as stochastic user equilibrium models (e.g., Sheffi, 1985), risk-taking models (e.g., Mirchandani and Soroush, 1987; Yin and Ieda, 2001), travel time budget models (e.g., Lo et al., 2006; Shao et al., 2006). The purpose of this paper is to offer another alternative model that enriches the literature and to fully explore the properties and consequences of bounded rationality in route choices. More specifically, as Mahmassani and Chang (1987) point out and discussed below, BRUE flow distributions in a static network may not be unique and we characterize the set of all possible BRUE flow distributions as a non-convex and non-empty set. In one interpretation, the flow distribution we observe at one particular day is just a particular realization from the set. Such an uncertainty incurred by boundedly rational travel behaviors may deteriorate the effectiveness of traditional congestion pricing strategies. For example, the link-based marginal-cost (MC) pricing scheme commonly advocated in the congestion pricing literature may not necessarily reduce congestion to its minimum level because users may not switch to routes with the least generalized cost. In this paper, we seek a tolling pattern that minimizes the worst-case system travel delay among all the possible BRUE flow patterns based on generalized costs (time plus tolls).

For the remainder, Section 2 mathematically defines BRUE and discusses the property of the set of all possible BRUE flow patterns. The problems of finding the best- and worst-case system travel times among the possible BRUE flow patterns are formulated as mathematical programs with complementarity constraints (MPCC) and then illustrated with examples. Section 3 formulates congestion pricing models that minimizes the worst-case system travel time and develops a heuristic solution algorithm to the models. Numerical examples are then presented to demonstrate and validate the models and solution algorithm. Finally, Section 4 concludes the paper.

2. Boundedly rational user equilibrium in a static network

2.1. Boundedly rational flow distribution and equilibrium conditions

As explained in Mahmassani and Chang (1987) and Chen et al. (1997), travelers with bounded rationality still follow the behavior that exhibits a tendency toward utility maximization, but not necessarily to the absolute maximum level. We thus define travelers with bounded rationality as those who (a) always choose routes with no cycle and (b) do not necessarily switch to the shortest (cheapest) routes when the difference between the travel times (or costs) on their current routes and the shortest one is no larger than a threshold value.

Using the terminology in Ahuja et al. (1993), all utilized routes under bounded rationality must correspond to paths. In the literature, some prefer to add the adjective "simple" and refer to routes with no cycle as "simple paths." (Ahuja et al., 1993), refer to routes with cycles as "walks.") Below are formal definitions of acceptable paths and BRUE:

Definition 2.1. A path is "acceptable" if the difference between its travel time or cost and that of the shortest or least-cost path is no larger than a pre-specified threshold value.

Definition 2.2. A path flow distribution is in BRUE if it is feasible or compatible with the travel demands and every user uses an acceptable path.

Unlike the conventional or perfectly rational user equilibrium (PRUE), the flow distribution under BRUE as defined above may not utilize any shortest or least-cost path.

To mathematically define BRUE, let W denote the set of O–D pairs and q^w represent travel demand for O–D pair $w \in W$. Let P^w be the set of paths for O–D pair w. For each path $r \in P^w$, f_r^w and c_r^w denote the corresponding path flow and path travel time. We further use f^w to represent a vector of path flows for O–D pair w, with f_r^w as its elements, and $f = (f^1, \ldots f^w, \ldots, f^{|W|})^T$, where $|\bullet|$ denotes the cardinality of a set. Similarly, c^w is a vector of path travel times for O–D pair w and c is a vector of c^w. Assuming that users of the same O–D pair have the same threshold value, denoted as $\bar{\varepsilon}^w$, f is a BRUE distribution if and only if there exists λ^w for every w such that

$$\begin{aligned}
&c_r^w \geq \lambda^w \quad \forall r \in P^w \\
&c_r^w \leq \lambda^w + \bar{\varepsilon}^w \quad \forall r \in P_+^w = \{r : f_r^w > 0, r \in P^w\} \\
&\sum_{r \in P^w} f_r^w = q^w \\
&f_r^w \geq 0 \quad \forall r \in P^w
\end{aligned}$$