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Equilibrium analysis of macroscopic traffic oscillations

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ABSTRACT

Using a simple network model with two parallel links connecting a diverge and a merge, this paper studies under what conditions traffic oscillations may be initiated and propagated in a traffic stream, specially at freeway bottlenecks. Drivers are assumed to minimize either the *experienced* or *instantaneous* travel times, and in doing so, they settle at a Wardrop (day-to-day) equilibrium or a Boston (within-day) traffic equilibrium, respectively. We prove that the path travel time function in our model is not monotone, and show that this property leads to multiple Wardrop equilibria, of which only one is both *stable* and *efficient*. The paper shows that periodic traffic oscillations do not arise from Wardrop equilibria. Trivial oscillations exist at Boston equilibria, which are caused by drivers' overreaction to traffic conditions. However, periodic oscillations are likely to emerge when (1) transitions between stable and unstable equilibria take place, and more importantly, (2) drivers make decisions based on out-of-date information of traffic conditions. The latter finding is useful in guiding control practice at freeway bottlenecks and work zones to prevent traffic oscillations.

1. Introduction

Traffic oscillations often arise in congested traffic flow, such as vehicular queues induced by freeway bottlenecks (for recent empirical evidences, see e.g., Smilowitz et al., 1999; Mauch and Cassidy, 2002; Ahn and Cassidy, 2007). In the past, this frustrating *stop-and-go* motion is often explained using car-following behavior (e.g. Chandler et al., 1958; Herman et al., 1959; Treiterer and Myers, 1974), lane-changing maneuvers (e.g. Gazis et al., 1962; Munjal and Pipes, 1971; Daganzo, 2002a,b), and the instability predicted by higher-order traffic flow models (e.g. Kerner and Konhauser, 1994; Jin and Zhang, 2003).

Traffic oscillations may also be triggered by *macroscopic* mechanisms such as queue interactions (Jin, 2003). Recently, Nie and Zhang (2008) and Jin (2009) characterized this type of oscillatory traffic pattern using a two-route network with a diverge and a merge (hereafter referred to as the D–M model) (see Fig. 1). These studies employ the traffic flow model of Lighthill and Whitham (1955) and Richards (1956) whereas the merge and diverge traffic follow the models of Jin and Zhang (2001) and Daganzo (1995), respectively. Periodic oscillations may occur in this model when queues formed at the merge spill back to the diverge, thereby reducing the discharging capacity of the diverging branches due to the first-in–first-out (FIFO) discipline imposed at the diverge. Noticeably, key features of such oscillatory traffic patterns appear to agree with empirical evidence such as reported in Mauch and Cassidy (2002) and Ahn and Cassidy (2007). This coincidence raises an interesting question of whether such a model can be used to explain, if not predict, traffic oscillations often observed at freeway bottlenecks. Valid though that question may seem, it should be noted that the original

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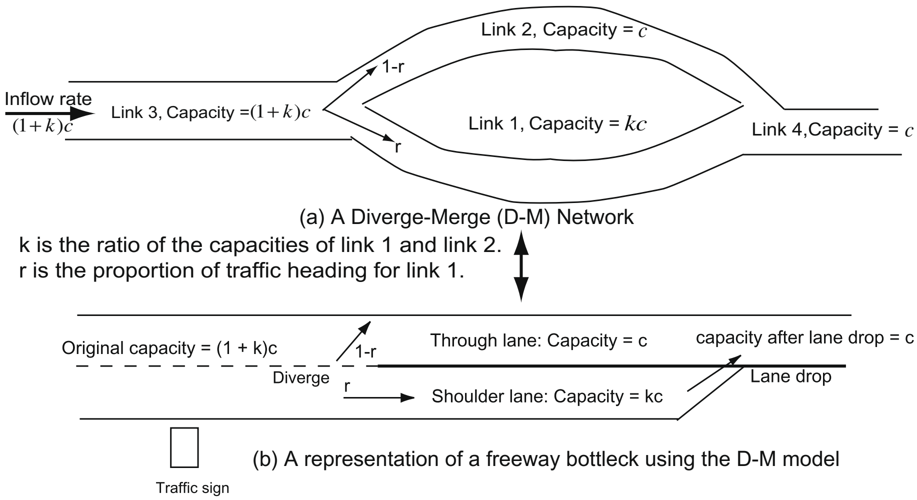

Fig. 1. A two-route network with a merge and a diverge.

D–M model ignores drivers' route choice behavior, namely, oscillations occur only when flow distribution at the diverge is fixed within a certain range. Obviously, unless an effective control device is implemented in its favor, drivers may not follow such a fixed route choice.

Would traffic oscillations emerge when drivers' behavior is reasonably taken into consideration? The present paper is intended to address this question. Well-embraced behavioral assumptions state that drivers tend to make travel choices (departure time, routes, etc.) to maximize their utility. For the purpose of this study, it suffices to focus on route choice and assume that travel time is the only factor at work in that choice. It is well-known that the traffic assignment problem (the problem of assigning traffic to shortest routes) with such behavioral assumptions can be formulated as a Nash–Cournot non-cooperative game, whose solutions are characterized by a set of traffic equilibrium states. Since the temporal evolution of traffic flow has to be considered in order to study oscillations, our equilibrium analysis falls into a class of *dynamic traffic assignment* (DTA) models. The reader is referred to Peeta and Ziliaskopoulos (2001) for a comprehensive review of the DTA literature.

Our goal is to study the properties of dynamic equilibrium solutions to the diverge–merge network model illustrated in Fig. 1a. Such a simple model allows us to derive analytical solutions that promise useful insights. More importantly, the D–M model reasonably represents a bottleneck situation where lane drop may cause vehicular queues and subsequently traffic oscillations, as shown in Fig. 1b. Assumptions necessary to make such a connection are:

- Drivers are informed of the bottleneck by a traffic sign upstream of the lane drop. In response to this event, drivers will make a lane-changing decision at *a point* near the sign. That point corresponds to the diverging junction in Fig. 1a. The ratio of drivers who select the shoulder lanes (i.e., link 1 in Fig. 1a) is denoted as r.
- Once passing the imaginary diverge, drivers will not change lane until they arrive at the actual lane-drop location, which constitutes the merging junction in Fig. 1a.

The settings in Fig. 1 will be frequently used hereafter. Particularly, we emphasize that links 1 and 2 in Fig. 1a refer to the shoulder lanes and through lanes in Fig. 1b, respectively.

Two different behavioral assumptions, which lead to different equilibrium states, are considered. In the first, drivers want to minimize their *experienced* travel times. By learning from and adjusting according to daily travel experience, drivers will settle at the so-called day-to-day equilibrium, which is a dynamic extension of the Wardrop equilibrium (Wardrop, 1952) and is widely used for long-term travel forecasting (e.g. Smith, 1993; Friesz et al., 1993; Ran et al., 1996). However, lane-changing maneuvers as those triggered by a lane drop in Fig. 1b may be too minor to be predicted from such a day-to-day equilibrium. It is more likely that drivers would make those lane-changing decisions *en-route* according to local traffic conditions. This assumption drives the system to a Boston traffic equilibrium (Friesz et al., 1993), in which drivers minimize their *instantaneous* travel times. The focus of the paper is, therefore, to obtain both Wardrop and Boston traffic equilibria of the D–M model and reveal their analytical properties, particularly those pertinent to oscillations. Numerical experiments will be conducted when it is difficult to get simple analytical solutions.

This paper is organized as follows. Section 2 briefly reviews the oscillatory traffic patterns yielded from the D–M model when the route choice is fixed. Sections 3 and 4 discuss the Wardrop and Boston traffic equilibria, respectively. Section 5 concludes the paper.