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Evidence of convective instability in congested traffic flow: A systematic empirical and theoretical investigation

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ABSTRACT

An extended open system such as traffic flow is said to be convectively unstable if perturbations of the stationary state grow but propagate in only one direction, so they eventually leave the system. By means of data analysis, simulations, and analytical calculations, we give evidence that this concept is relevant for instabilities of congested traffic flow. We analyze detector data from several hundred traffic jams and propose estimates for the linear growth rate, the wavelength, the propagation velocity, and the severity of the associated bottleneck that can be evaluated semi-automatically. Scatter plots of these quantities reveal systematic dependencies. On the theoretical side, we derive, for a wide class of microscopic and macroscopic traffic models, analytical criteria for convective and absolute linear instabilities. Based on the relative positions of the stability limits in the fundamental diagram, we divide these models into five stability classes which uniquely determine the set of possible elementary spatiotemporal patterns in open systems with a bottleneck. Only two classes, both dominated by convective instabilities, are compatible with observations. By means of approximate solutions of convectively unstable systems with sustained localized noise, we show that the observed spatiotemporal phenomena can also be described analytically. The parameters of the analytical expressions can be inferred from observations, and also (analytically) derived from the model equations.

1. Introduction

Traffic flow dynamics and congestion phenomena have been investigated for decades (Reuschel, 1950; Pipes, 1953) (see also the reviews (Helbing, 2001; Hoogendoorn and Bovy, 2001)). Observations of instabilities date back several decades as well (Treiterer and Myers, 1974), and there is an ongoing flow of new empirical results (Kerner and Rehborn, 1996; Zielke et al., 2008; Schönhof and Helbing, 2007; Treiber and Kesting, submitted for publication). Some fundamental questions, however, are not yet settled. Can the multitude of observed spatiotemporal patterns of congested traffic be decomposed into precisely defined elementary patterns (Kerner and Rehborn, 1996; Schönhof and Helbing, 2009; Treiber et al., 2010a)? If so, into how many patterns, and what are their defining properties (Schönhof and Helbing, 2007; Kerner, 2002)? What is the relation between traffic patterns and (dynamic) traffic phases (Treiber et al., 2010a)? Particularly, are there two or three phases (Wagner and Nagel, 2008)?

All this is related to oscillations in extended regions of congested traffic, i.e., congested traffic spreading over several detector locations. Such oscillations have been observed frequently, e.g., in Germany, Holland, England, and the USA (Zielke

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et al., 2008; Bertini and Leal, 2005; Ahn and Cassidy, 2007; Helbing et al., 2009; Wilson, 2008; Treiber et al., in press). Their common properties have been summarized by the following *qualitative facts* (Treiber et al., 2010a).

(1) Congestion patterns are typically caused by bottlenecks. The downstream front is either stationary at a bottleneck, or moves at a constant velocity c_{cong}. Analyzing about 400 congestion patterns on the German freeways A5-North and A5-South did not bring conclusive evidence of a single breakdown without a bottleneck (Schönhof and Helbing, 2007).
(2) Most extended traffic patterns on freeways exhibit distinct internal oscillations (Daganzo, 2002a; Mauch and Cassidy, 2002). Only minor or severe bottlenecks may cause nearly homogeneous congested traffic where possible oscillations cannot be distinguished unambiguously from noise (Treiber et al., 2010a).
(3) The oscillations propagate in upstream direction at the characteristic velocity c_{cong} of the moving downstream fronts (cf. Fact 1) (Li et al., 2010; Orosz et al., 2010; Windover and Cassidy, 2001). This velocity depends weakly on the country and traffic composition (Zielke et al., 2008; Bertini and Leal, 2005; Ahn and Cassidy, 2007), but not on the type of congestion.
(4) The amplitude of the oscillations increases while propagating upstream (Zielke et al., 2008). As a consequence, one observes stationary traffic near the bottleneck and growing stop-and-go waves further upstream.
(5) If oscillations are present, their frequency increases with the severity of the bottleneck.

We emphasize that, for the sake of simplicity, we do not consider details of the bottleneck which may be caused by onramps, offramps, gradients, road works, accidents, or combinations thereof. Nor do we consider the immediate region of a bottleneck itself where the dynamics generally depends on the details of the activation mechanism. For example, on US freeways with many lanes, lane separation effects such as the "rabbits and slugs" phenomenon (Daganzo, 2002b) may be relevant in the vicinity of the bottleneck. In contrast, in our data base, we always have observed nearly perfect speed synchronization across all lanes.

Of course, any "theory" must be consistent with the above observations. However, these observations do not settle unambiguously some of the most intriguing questions about the nature of traffic instabilities: Are they of a nonlinear type requiring a finite perturbation for activation, or are the observations consistent with linear instabilities? Can insights into stability properties of homogeneous systems (or closed ring roads) be transferred to real open systems (Helbing et al., 1999; Helbing et al., 2009; Treiber et al., 2010a)? Can the above observations be "explained" quantitatively and analytically in terms of the convective instability? These questions are clearly fundamental since instabilities are the main building block for dynamic traffic phases and spatiotemporal patterns (Wagner and Nagel, 2008; Treiber et al., 2010a).

In this paper we investigate the problem from two perspectives. On the empirical side, we analyze detector data of several hundred traffic jams and quantify the above *qualitative facts* with estimates that can be evaluated semi-automatically. The main part of the paper is dedicated to a theoretical analysis of these results. After providing analytic criteria for the thresholds of absolute and convective linear instabilities for a wide spectrum of microscopic and macroscopic traffic flow models, we divide the models into five classes which uniquely determine the set of observable spatiotemporal patterns in real systems. Both simulations and analytical solutions show that only two of them are consistent with observations. Our main result is that the largely neglected concept of *convective instability* (Huerre and Monkewitz, 1990) which only very recently has attracted more attention in the traffic flow community (Ward and Wilson, in press; Wilson and Ward, 2011) has a high explanative power in describing the observed spatiotemporal dynamics of extended traffic jams.

The paper is structured as follows: Section 2 presents a scheme to extract quantitative aspects of the spatiotemporal evolution of congested traffic patterns from cross-sectional data. In order to provide input for the theory to be developed, the scheme is applied to a database containing several hundred instances of jams. Section 3 gives analytical criteria for limits of absolute and convective string instability that are applicable to a wide range of microscopic and macroscopic traffic models. Moreover, the relative locations of the stability limits are connected to the observed patterns by a newly formulated "class diagram" in parameter space. As an example, the full parameter space of the Intelligent Driver Model (Treiber et al., 2000) is explored. Section 4 gives approximate analytic solutions to the equations of the considered models for realistic congested situations, i.e., open systems with local sustained noise that may represent lane changes at bottlenecks. The implications of these results are discussed in Section 5.

2. Extracting quantitative properties of wave propagation from traffic data

In this section, we propose quantitative estimates for the wavelength, propagation velocity and growth rate of oscillations in congested traffic. Since these quantities depend on the severity of the bottleneck, we give data-driven estimates for the "bottleneck strength" as well. Here, we will only give a summary; details are published elsewhere (Treiber and Kesting, submitted for publication).

2.1. The proposed scheme

We assume that aggregated data for the flow Q and speed V (arithmetic averages) are available from several consecutive detector cross sections i at locations x_i, counted in the direction of traffic flow. Specifically, Q_{it} denotes the lane-averaged flow for cross section i during the tth time interval, and V_{it} denotes the corresponding speed.