Transportation Research Part B 45 (2011) 232–254

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Discretised link travel time models based on cumulative flows: Formulations and properties

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ARTICLE INFO

Article history:
Received 19 June 2009
Received in revised form 8 May 2010
Accepted 10 May 2010

Keywords:
Link travel time
Cumulative flow
Piecewise linearization
FIFO
Causality

ABSTRACT

In the research area of dynamic traffic assignment, link travel times can be derived from link cumulative inflow and outflow curves which are generated by dynamic network loading. In this paper, the profiles of cumulative flows are piecewise linearized. Both the step function (SF) and linear interpolation (LI) are used to approximate cumulative flows over time. New formulations of the SF-type and LI-type link travel time models are developed. We prove that these two types of link travel time models ensure first-in-first-out (FIFO) and continuity of travel times with respect to flows, and have other desirable properties. Since the LI-type link travel time model does not satisfy the causality property, a modified LI-type (MLI-type) link travel time model is proposed in this paper. We prove that the MLI-type link travel time model ensures causality, strong FIFO and travel time continuity, and that the MLI-type link travel time function is strictly monotone under the condition that the travel time of each vehicle on a link is greater than the free flow travel time on that link. Numerical examples are set up to illustrate the properties and accuracy of the three models.

1. Introduction

The properties of dynamic traffic assignment (DTA) have important implications on its ability to portray the actual travel behavior and computation speed. These properties strongly depend on two components of DTA (Szeto and Lo, 2006): the travel choice principle and the traffic flow component.

The travel choice principle in DTA models travelers' propensity to travel, e.g., how they select their routes, departure times, modes, or destinations. In making such choices, travel time is one important element of their considerations. The dynamic user optimal (DUO) principle is in general adopted as the travel choice principle in DTA , which assumes that travelers select their routes and/or departure times to minimize their actual travel costs such as travel times. The travel choice principle can be mathematically formulated as a variational inequality problem (e.g., Friesz et al., 1993; Ran and Boyce, 1996; Lo and Szeto, 2002; Huang and Lam, 2002), where link travel times are functions of link flows. The existence of solutions to this problem requires the mapping function of the problem to be continuous and the solution set to be a nonempty compact convex set while the uniqueness of the solution further requires the mapping function to be strictly monotonic (Nagurney, 1993). These requirements imply that link travel times must be continuous with respect to link flows for solution existence;

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0191-2615/$ - see front matter
doi:10.1016/j.trb.2010.05.002

moreover, link travel times must be strictly monotone with respect to link flows for solution uniqueness. If the continuity cannot be guaranteed, the DTA problem can have no solution and the solution algorithms cannot give a convergent solution. Furthermore, if the travel time function is monotone, DTA can be solved by some existing solution methods efficiently. Therefore, continuity and monotonicity of travel times are two important properties of DTA.

The traffic flow component depicts how traffic propagates inside a traffic network and hence governs the network performance in terms of travel time. The procedure in implementation is often called dynamic network loading (DNL). In developing such component, one approach is to develop link travel time models. Link travel time models often express the travel time of a link as a function of the flow on that link. The link travel time models presented in the literature generally focus on the following properties: first-in-first-out (FIFO) (e.g., Astarita, 1996; Huang and Lam, 2002; Carey et al., 2003; Carey and Ge, 2005, 2007), causality (e.g., Friesz et al., 1993; Astarita, 1996; Carey et al., 2003; Carey and Ge, 2007), and reduction to a static model (e.g., Carey et al., 2003; Carey and Ge, 2007). FIFO implies that vehicles that enter the link earlier will leave it sooner. Causality means that the speed and travel time of a vehicle on a link is only affected by the speed of vehicles ahead. Reduction to a static model means the link travel time function should reduce to the well-known static version when traffic flows are constant over time. Indeed, FIFO and causality are two actual traffic behaviors. A dynamic link travel time model is necessary to satisfy FIFO and causality in order to obtain the solutions of DTA that are consistent with actual traffic behavior.

The second approach to develop the travel flow component is based on either exit flow functions (e.g., Merchant and Nemhauser, 1978; Carey and Srinivasan, 1993; Lam and Huang, 1995; Wie and Tobin, 1998; Shin et al., 2000) or advanced exit flow functions (e.g., Kuwahara and Akamatsu, 2001; Lo and Szeto, 2002; Lian et al., 2007; Yperman, 2007; Szeto, 2008; Nie and Zhang, 2010). The exit flow function approach treats the outflow of a link or a segment of link as a non-decreasing function of the number of vehicles on the whole link or the link segment, respectively. The advanced exit flow functions are developed based on either Daganzo's (1994, 1995a) solution scheme (referred to as the cell transmission model (CTM)) or Newell's (1993) solution scheme to the Lighthill and Whitham (1955) and Richards (1956) (LWR) hydrodynamic model of traffic flow. The main difference between these functions and the exit flow functions is that the advanced exit flow functions consider storage capacity to capture the effects of physical queues like queue spillback. If link inflow rates are given, the cumulative outflows can be generated by a DNL model. No matter whether the exit flow functions or the advanced exit flow functions are used for the DNL model, after the outflow and inflow rates of each link are determined, the cumulative flows and hence travel times can be obtained.

Deriving travel times from cumulative flow curves is a fundamental step in the algorithms of many DTA models, and hence it is important to develop accurate and efficient method to do this. For example, the differences between policy scenarios studied using DTA models are often small. Therefore, it is important that other approximation or discretization errors must be kept even smaller relative to those. Indeed, discretization of time is needed to obtain travel time and even solve continuous time DTA models, since there are no known methods for solving complex continuous time models analytically. The outcome is that even if the cumulative curves have desirable properties (e.g., FIFO, causality, etc), the fineness of time discretization affects whether the discretised model (and in particular the travel time functions) retain these desirable properties. One may use a fine discretization to try to retain these properties so that the DNL is consistent with travel time estimation and that the overall model is theoretically sound. Nevertheless, our experience, and that of many authors worked with many DTA models and algorithms are that it is not computationally tractable to use very fine discretizations, as it can take excessive amounts of time even for medium or small networks. It is therefore important to investigate (i) whether the discretised travel time functions derived from cumulative flow curves with desirable properties can retain these properties, and (ii) how this is affected by the fineness of time discretization. However, in the literature, travel times derived from cumulative flows are calculated according to their proposed methods, and there are no travel time formulations derived from these flows for analysis of the properties of travel time.

This paper develops three link travel time formulations for DTA based on cumulative flows, which are different from traditional link travel time models that formulate travel time as a function of time-varying link flows (e.g., Daganzo, 1995b; Carey et al., 2003; Carey and Ge, 2007). Two of the three formulations are reformulated from existing travel time models for analysis purposes and the remaining formulation is new. Each cumulative flow profile is approximated by either a step function (SF) or linear interpolation (LI). In particular, the first formulation, namely the SF-type formulation, approximates the profiles of link cumulative inflows and outflows by step functions whereas the other two formulations, namely, the LI-type and the proposed modified LI-type (MLI-type) formulations, approximate the profiles by linear interpolations. The proposed formulations allow us to analyse the properties of the corresponding travel time functions including continuity, monotonicity, FIFO, and causality. Moreover, the accuracy of travel times derived from cumulative flow curves is unknown until this study.

This paper also discusses the properties of the three novel formulations. Note that the property of reduction to a static model is not considered in this paper but left for future research. For clarity, we restate the definitions of FIFO and causality discussed by Carey et al. (2003):

Property 1 (FIFO). *This can be stated in various equivalent ways, for example, traffic that enters a link up to any time t will exit from the link before traffic that enters after time t. This is not intended to preclude individual vehicles, traveling in the same direction, overtaking and passing each other. Hence, for traffic entering at say time t, the exit time can be interpreted as an 'average' exit time, or the exit time for an 'average' vehicle.*