Available online at www.sciencedirect.com

ScienceDirect

Transportation Research Part B 42 (2008) 274–303

TRANSPORTATION
RESEARCH
PART B

www.elsevier.com/locate/trb

The multiple discrete-continuous extreme value (MDCEV) model: Role of utility function parameters, identification considerations, and model extensions

Chandra R. Bhat *,[1]

The University of Texas at Austin, Department of Civil, Architectural & Environmental Engineering, 1 University Station C1761, Austin, TX 78712-0278, USA

Received 1 June 2006; received in revised form 6 June 2007; accepted 7 June 2007

Abstract

Many consumer choice situations are characterized by the simultaneous demand for multiple alternatives that are imperfect substitutes for one another. A simple and parsimonious multiple discrete-continuous extreme value (MDCEV) econometric approach to handle such multiple discreteness was formulated by Bhat (2005) [Bhat, C.R., 2005. A multiple discrete-continuous extreme value model: formulation and application to discretionary time-use decisions. Transportation Research Part B 39(8), 679–707]. within the broader Kuhn–Tucker (KT) multiple discrete-continuous economic consumer demand model of Wales and Woodland (1983) [Wales, T.J., and Woodland, A.D., 1983. Estimation of consumer demand systems with binding non-negativity constraints. Journal of Econometrics 21(3), 263–85]. This paper examines several issues associated with the MDCEV model and other extant KT multiple discrete-continuous models. Specifically, the paper proposes a new utility function form that enables clarity in the role of each parameter in the utility specification, presents identification considerations associated with both the utility functional form as well as the stochastic nature of the utility specification, extends the MDCEV model to the case of price variation across goods and to general error covariance structures, discusses the relationship between earlier KT-based multiple discrete-continuous models, and illustrates the many technical nuances and identification considerations of the multiple discrete-continuous model structure through empirical examples. The paper also highlights the technical problems associated with the stochastic specification used in the KT-based multiple discrete-continuous models formulated in recent Environmental Economics papers.

Keywords: Discrete-continuous system; Multiple discreteness; Kuhn–Tucker demand systems; Mixed discrete choice; Random utility maximization

* Tel.: +1 512 471 4535; fax: +1 512 475 8744.
E-mail address: bhat@mail.utexas.edu

[1] This paper was completed when the author was a Visiting Professor at the Institute of Transport and Logistics Studies, Department of Economics and Business, University of Sydney.

0191-2615/$ - see front matter
doi:10.1016/j.trb.2007.06.002

1. Introduction

Multiple discreteness (i.e., the choice of multiple, but not necessarily all, alternatives simultaneously) is a rather ubiquitous characteristic of consumer decision-making.[2] Examples of multiple discreteness include situations where an individual may decide to participate in multiple kinds of maintenance and leisure activities within a given time period (Bhat, 2005), or a household may own a mix of different kinds of vehicles (such as a sedan and a pick-up truck or a sedan and a minivan; see Bhat and Sen, 2006). Such multiple discrete situations may be modeled using the traditional random utility-based (RUM) single discrete choice models by identifying all combinations or bundles of the "elemental" alternatives, and treating each bundle as a "composite" alternative (the term "single discrete choice" is used to refer to the case where a decision-maker chooses only one alternative from a set of alternatives). A problem with this approach, however, is that the number of composite alternatives explodes with the number of elemental alternatives. Another approach is to use the multivariate probit (logit) methods of Manchanda et al. (1999), Baltas (2004), Edwards and Allenby (2003), Bhat and Srinivasan (2005). But this approach is not based on a rigorous underlying utility-maximizing framework of multiple discreteness; rather, it represents a statistical "stitching" of univariate utility maximizing models. In both the approaches discussed above to handle multiple discreteness, there is also no explicit way to accommodate the diminishing marginal returns (i.e., satiation) in the consumption of an alternative. Additionally, and related to the above point, it is very cumbersome, even if conceptually feasible, to include a continuous dimension of choice (for example, modeling the durations of participation in the chosen activity purposes, in addition to the choice of activity purpose).[3]

Wales and Woodland (1983) proposed two alternative ways to handle situations of multiple discreteness within a behaviorally-consistent utility maximizing framework. Both approaches assume a direct utility function $U(\boldsymbol{x})$ that is assumed to be quasi-concave, increasing, and continuously differentiable with respect to the consumption quantity vector $\boldsymbol{x}$.[4] Consumers maximize the utility function subject to a linear budget constraint, which is binding in that all the available budget is invested in the consumption of the goods; that is, the budget constraint has an equality sign rather than a '$\leqslant$' sign. This binding nature of the budget constraint is the result of assuming an increasing utility function, and also implies that at least one good will be consumed. The difference in the two alternative approaches proposed by Wales and Woodland (1983) is in how stochasticity, non-negativity of consumption, and corner solutions (i.e., zero consumption of some goods) are accommodated, as briefly discussed below (see Wales and Woodland, 1983; Phaneuf et al., 2000 for additional details).

The first approach, which Wales and Woodland label as the Amemiya–Tobin approach, is an extension of the classic microeconomic approach of adding normally distributed stochastic terms to the budget-constrained utility-maximizing share equations. In this approach, the direct utility function $U(\boldsymbol{x})$ itself is assumed to be deterministic by the analyst, and stochasticity is introduced post-utility maximization. The justification for

[2] A brief history of the term "multiple discreteness" is in order here. Traditional discrete choice models focus on the selection of a single alternative from the set of available alternatives on a purchase occasion. That is, they consider the "extreme corner solution problem". Hanemann, in his 1978 dissertation, used the term "generalized corner solution problem" to refer to the situation where multiple alternatives may be chosen simultaneously. Hendel (1999) appears to have been the first to coin the term "multiple discreteness" to refer to the choice of multiple alternatives. This term is also used by Dube (2004).

[3] Another approach for multiple discreteness is the one proposed by Hendel (1999) and Dube (2004). These researchers consider the case of "multiple discreteness" in the purchase of multiple varieties within a particular product category as the result of a stream of expected (but unobserved to the analyst) future consumption decisions between successive shopping purchase occasions (see also Walsh, 1995). During each consumption occasion, the standard discrete choice framework of perfectly substitutable alternatives is invoked, so that only one product is consumed. Due to varying tastes across individual consumption occasions between the current shopping purchase and the next, consumers are observed to purchase a variety of goods at the current shopping occasion. A Poisson distribution is assumed for the number of consumption occasions and a normal distribution is assumed regarding varying tastes to complete the model specification. Such a "vertical" variety-seeking model, of course, is different from the "horizontal" variety seeking model considered in this paper, where the choice is considered to be among inherently imperfect substitutes at the choice occasion (see Kim et al., 2002; Bhat, 2005).

[4] The assumption of a quasi-concave utility function is simply a manifestation of requiring the indifference curves to be convex to the origin (see Deaton and Muellbauer, 1980, p. 30 for a rigorous definition of quasi-concavity). The assumption of an increasing utility function implies that $U(\boldsymbol{x}^1) > U(\boldsymbol{x}^0)$ if $\boldsymbol{x}^1 > \boldsymbol{x}^0$.