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The traffic equilibrium problem with nonadditive costs and its monotone mixed complementarity problem formulation

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Abstract

Various models of traffic equilibrium problems (TEPs) with nonadditive route costs have been proposed in the last decade. However, equilibria of those models are not easy to obtain because the variational inequality problems (VIPs) derived from those models are not monotone in general. In this paper, we consider a TEP whose route cost functions are nonadditive disutility functions of time (with money converted to time). We show that the TEP with the disutility functions can be reformulated as a monotone mixed complementarity problem (MCP) under appropriate conditions. We then establish the existence and uniqueness results for an equilibrium of the TEP. Numerical experiments are carried out using various sample networks with different disutility functions for both the single-mode case and the case of two different transportation modes in the network.

Keywords: Traffic equilibrium problem; Variational inequality problem; Mixed complementarity problem; Monotone; Nonadditive cost

1. Introduction

In the study of the traffic equilibrium problem (TEP), the researchers have presented various formulations in which many different assumptions are made to represent the ‘‘real’’ traffic conditions (Aashtiani and Magnanti, 1981, Chen et al., 1999, Dafermos, 1980). One of the standard assumptions used is that *the route costs faced by the users in the network are* ***additive***. That is, *the route costs are simply the sum of the arc costs for all the arcs on the route being considered.*

There are many situations, however, where this additivity assumption on the route costs is inappropriate. Gabriel and Bernstein (1997) discussed some of the situations where nonadditive route costs occur. They claimed that almost all toll and fare schemes being implemented around the world are nonadditive. For example, the different pricing policies such as congestion pricing and the collection of emission fees add to the

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nonadditivity of travel costs. Moreover, different individuals have different valuations of time, which contributes to the nonadditivity of route costs.

Although nonadditivity is important in presenting a more realistic view of the traffic situation, it causes a difficulty in the analysis and computation of an equilibrium, which are usually done by formulating the TEP as the variational inequality problem (VIP). The VIP is generally stated as follows (Facchinei and Pang, 2003): Find a vector $x \in K$ such that

$$(y - x)^{\mathrm{T}} G(x) \geqslant 0 \quad \forall y \in K, \tag{1.1}$$

where K is a nonempty closed convex subset of $\Re^n$ and $G : K \to \Re^n$ is a continuous function. Special cases of the VIP include the nonlinear complementarity problem (NCP) and the mixed complementarity problem (MCP). The NCP is the VIP with $K = \Re^n_+ \equiv \{x \in \Re^n | x_i \geqslant 0,\ i = 1, \ldots, n\}$ and the MCP is the VIP with $K = \{x \in \Re^n | a_i \leqslant x_i \leqslant b_i,\ i = 1, \ldots, n\}$, where $a_i \in \Re \cup \{-\infty\}$, $b_i \in \Re \cup \{+\infty\}$, $a_i \leqslant b_i$, $i = 1, \ldots, n$. We denote the NCP with the function G by NCP(G) and the MCP with the function G and the set K by MCP(G, K).

In the last decades, the VIP has been studied extensively. The monotonicity of G particularly plays an important role in the existence and uniqueness of solutions of VIP. Moreover, the monotonicity is also important for solution methods for VIP to work efficiently. We recall that a function $G : \Re^n \to \Re^n$ is called

(i) *monotone* if $(x - y)^{\mathrm{T}}(G(x) - G(y)) \geqslant 0 \quad \forall\, x, y \in \Re^n$; and
(ii) *strictly monotone* if $(x - y)^{\mathrm{T}}(G(x) - G(y)) > 0 \quad \forall x, y \in \Re^n (x \neq y)$.

We also say that VIP or NCP or MCP is monotone if G is monotone. Most of the existing results for the VIP rely on the assumption that the function G involved satisfies certain conditions such as strong or strict monotonicity (Facchinei and Pang, 2003).

A VIP equivalent to the TEP with additive costs may usually be formulated as a monotone VIP (Facchinei and Pang, 2003). However, a VIP derived from the TEP with nonadditive costs does not immediately possess monotonicity unless restrictive assumptions are made or a certain reformulation is introduced.

Lo and Chen (2000) considered a special case of the TEP with nonadditive cost functions. Specifically, they introduced a route-specific cost structure, where the route cost is assumed to be the sum of the travel time and an additional charge which is route-specific (a specific travel cost, possibly in the form of toll, is added only to a particular route in the network). This additional cost is only incurred by travelers on that route. They showed that the equivalent NCP becomes monotone. However, they reported that other users of the network (not necessarily using this route) are affected by this added route cost when they share a common link with the route with the added cost. Moreover, the route cost function they considered was very simple, hence not so realistic. In order to solve the TEP, they converted the NCP formulation into an equivalent optimization problem by using a merit function.

Gabriel and Bernstein (1997) proposed a more general route cost function. They also used some assumptions on the route costs in order to ensure monotonicity of their formulation. However, as will be shown in Section 2.3, those assumptions imply that the cost function is an affine function of time. In their work, they proposed a merit function approach to solve the NCP formulation of the TEP with nonadditive costs. Their method was based on transforming the NCP first into a problem of finding a zero of a system of nonsmooth equations. The problem can be solved by using an existing method when the NCP is monotone.

In this paper, we modify the model presented by Gabriel and Bernstein (1997) by introducing a disutility function. We show that the equivalent VIP can be transformed into a monotone MCP, and then give the existence and uniqueness results for the proposed model.

This paper is organized as follows. In the following section, we provide an overview of the important concepts used in this paper, namely, the traffic equilibrium principle, the MCP formulation of the TEP, and the nonadditive travel costs. The proposed TEP and its monotone MCP reformulation are presented in Section 3. We also establish the existence and uniqueness results in this section. Computational results for TEPs with different disutility functions and various networks to compare our reformulation to the original VIP formulation are given in Section 4. We give a brief conclusion in Section 5.