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Economic order quantity and sampling inspection plans for imperfect items

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ABSTRACT

The creation of economic order quantity (EOQ) models that consider imperfect items is a step towards making these models even closer to real-world situations. The underlying assumption of these models, which is that conducting a full inspection to separate perfect items from imperfect ones, is a costly process and not always the best decision. In this paper, sampling inspection plans are designed on the basis of which a buyer would be able to make a decision regarding the next step. More precisely, we can find two cut-off points for the imperfect rate, and the best strategy is (i) not to conduct a full inspection when we expect to have imperfect rate below the first cut-off point; (ii) to conduct a full inspection when we expect to have an imperfect rate between the two cut-off points, and (iii) to reject the lot when the imperfect rate is expected to be higher than the second cut-off point. We formulate the problem, and derive the expected EOQ and optimal total profit of the problem. Numerical examples are used to illustrate the model. The numerical examples show that by considering the sampling inspection plans, the buyer gains more profit compared to the existing individual approaches (traditional EOQ, and EOQ models with full inspection).

1. Introduction

It is now more than a century since Harris (1913) formulated the first mathematical model to find the optimal inventory, which is known as economic order quantity (EOQ). Basically, the problem is to calculate the quantity to be ordered by the buyer each time, such that the total costs of the problem, including holding costs and ordering costs, be minimized. In the years since then, the original model has extended in different directions. One of the branches of the model created most recently incorporates quality into the basic problem. While, in the traditional EOQ models, it is implicitly assumed that the received items are of perfect quality, the real world does not support such an assumption. Salameh and Jaber (2000) were among the first researchers to incorporate quality into EOQ models, making them closer to the real world. They considered a situation where an average *p* percent of all items being ordered are imperfect. It is assumed that, in practice, the buyer conducts a full inspection of the items to separate the imperfect items from the perfect ones. It is also assumed that the imperfect items are sold as a single batch at the end of the inspection process. Formulating the problem, the EOQ is derived. The original model (Salameh & Jaber, 2000), corrected by Goyal and Cárdenas-Barrón (2002), extended by Papachristos and Konstantaras (2006) with respect to the occurrence of shortage, and the objective function formulation, and extensively revisited by Maddah and Jaber (2008), has gained significant attention by the other researchers, mainly due to its simplicity and practical value.

Some researchers considered the possibility of occurring shortage and backordering in the model (Chang & Ho, 2010; Eroglu & Ozdemir, 2007; Hsu & Hsu, 2013a; Rezaei, 2005; Wee, Yu, & Chen, 2007). While the original model assumes that the imperfect items are sold in a different inventory situation, with a lower price compared to perfect ones, some considered situations where imperfect items are reworked and then sold as perfect items (Hayek & Salameh, 2001; Konstantaras, Goyal, & Papachristos, 2007; Shekarian, Jaber, Kazemi, & Ehsani, 2014). Most studies consider a 100% inspection conducted by the buyer when inspection rate, and inspection cost are assumed to be constant throughout the planning horizon. However, some researchers have examined other possibilities. For example, some have studied the effect of learning on inspection (Konstantaras, Skouri, & Jaber, 2012; Wahab & Jaber, 2010), while Rezaei and Salimi (2012) have questioned the buyer carrying out a 100% inspection. They showed that, in some cases, it is more beneficial for the buyer if the supplier conducts the inspection and in return increases the product price.

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Some also considered the possibility of errors occurring in the inspection process (Hsu & Hsu, 2013a, 2013b; Khan, Jaber, & Ahmad, 2014; Khan, Jaber, & Bonney, 2011). Skouri, Konstantaras, Lagodimos, and Papachristos (2014) considered an EOQ with imperfect items and backorder, where to control the supply process an "all or none" policy is considered. That is to say, a full inspection is first conducted and the received lot is accepted if there is no imperfect item in the lot, otherwise it is rejected. They formulated the problem as a mathematical model and proposed a solution approach to find the optimal order quantity and backorder such that the expected total cost is minimized. For more information on some other extensions of the model, we refer to the review paper Khan, Jaber, Guiffrida, and Zolfaghari (2011).

Reviewing existing literature reveals that several aspects of the original problem modeled by Salameh and Jaber (2000) have been studied. These studies have considered the execution of a 100% inspection, of course in order not to sell imperfect items to the customers, who expect to receive a perfect product, and there may otherwise be some costly consequences. However, it is very important to consider that inspection is also a costly process, which is why, we think, sampling could be a better alternative, at least in some situations. The main contribution of this study is to make a bridge between the traditional EOQ models and the 'EOQ models for imperfect items'. While the traditional EOQ models stand at one extreme of the continuum ignoring the existence of imperfect items and as a result conducting an inspection, the 'EOQ models with imperfect items' stand at the other extreme by conducting a full inspection regardless of the expected imperfect rate. This study shows that the optimal strategy is somewhere in between. In this study, we formulate the problem with the aim of finding the EOQ, considering the possibility of different scenarios (no inspection, full inspection, rejection), based on the outcome of sampling inspection plans.

The rest of this paper is organized as follows. Section 2 contains the mathematical formulation of the problem. In Section 3, numerical examples are presented and Section 4, finally, contains the conclusion and suggestions for future research.

2. Problem formulation

The following notations are used in this paper.

D	Demand rate per time unit
y	Order quantity
c	Purchasing price per unit
s	Selling price per perfect unit
v	Selling price per imperfect unit
h	Holding cost per unit per time unit
x	Inspection rate per time unit
p	Imperfect rate
$f(p)$	Probability density function of imperfect rate
$E[p]$	Expected imperfect rate
d	Screening (inspection) cost per unit
θ	Number of imperfect items in a sample of n items
n	Sample size
K	Fixed ordering cost
r	Return cost per unit
R	Reject penalty cost
T	Ordering cycle duration

Consider a situation where a buyer wants to decide on economic order quantity of a product, where the received order contains a p percentage of imperfects, with a known probability density function, $f(p)$. Upon receiving the lot, the buyer draws a sample of size n from the lot and based on the findings from the sample, one of the three following decisions is made:

- If the imperfect rate p is above a maximum limit, the lot is rejected, which means the supplier will have to send a lot of the same size containing no imperfect items.
- If the imperfect rate p is below the maximum limit and above the minimum limit, the lot is accepted. The buyer then conducts a full inspection. The imperfect items are separated from the perfect ones and used in another inventory situation.
- If the imperfect rate p is below a minimum limit, the lot is accepted and no inspection is conducted. In this case, it is assumed that the customers who are receiving an imperfect item will return it to the buyer and ask for a new item.

We first formulate the total revenue and EOQ of these three cases, without considering the sampling costs or issues related to sampling. Then, for the above-mentioned sampling strategy, we determine the minimum and maximum limits for the percentage imperfect p. Finally, considering the probability density function $f(p)$ of imperfect rate p, we calculate the expected total revenue and EOQ of the entire problem.

Case 1. The imperfect rate lies above the maximum limit.

In this case, where the imperfect rate lies above the maximum limit p_1, the buyer rejects the lot, and the supplier sends a new lot with the same size containing no imperfect items. Considering the lot size y and selling price s, the total revenue would be sy. The total costs consist of purchasing costs cy, ordering costs K, and holding costs $\frac{hy^2}{2D}$, so the total profit of this case is calculated as follows:

$$TP(y) = sy - cy - K - \frac{hy^2}{2D} \tag{1}$$

Dividing $TP(y)$ by the inventory cycle length $T = \frac{y}{D}$ gives the buyer's total profit per time unit, as follows:

$$TPU(y|p \geqslant p_1) = (s-c)D - \frac{KD}{y} - \frac{hy}{2} \tag{2}$$

Consequently, the optimal order quantity of this case would be:

$$y_R^* = \sqrt{\frac{2DK}{h}} \tag{3}$$

Case 2. The imperfect rate lies between the minimum and the maximum limits.

In this case where the imperfect rate lies between the minimum limit p_0 and the maximum limit p_1, the buyer accepts the lot. The buyer then conducts a full inspection. The imperfect items are separated from the perfect ones and used in another inventory situation. This case is in fact what has been proposed by Maddah and Jaber (2008). The total revenue of this case consists of total revenue of perfect items, $sy(1-p)$, and total revenue of imperfect items, vyp. The total costs of this case consist of ordering costs K, purchasing costs cy, inspection costs dy and holding cost $h\left(\frac{[y(1-p)]^2}{2D} + \frac{py^2}{x}\right)$. The total profit of this case is calculated as follows:

$$TP(y) = sy(1-p) + vyp - K - cy - dy - h\left(\frac{[y(1-p)]^2}{2D} + \frac{py^2}{x}\right) \tag{4}$$

Dividing $TP(y)$ by the expected inventory cycle length $E[T] = \frac{(1-E[p])y}{D}$, the buyer's expected total profit per time unit is calculated as follows: