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Synthetic charts to control bivariate processes with autocorrelated data

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ABSTRACT

In this study, we propose the use of simultaneous $\overline{X}$ charts to control bivariate processes with autocorrelated data. The first set of $\overline{X}$ charts is side-sensitive with regard to the same variable (SV $\overline{X}$ charts) and the second one is side-sensitive with regard to both variables (BV $\overline{X}$ charts). The Markov chain approach was used to obtain the steady-state properties of the $\overline{X}$ charts. In comparison with the standard synthetic T^2 chart, the SV and the BV charts signal faster in a wide variety of disturbances, except when the variables are high correlated. The BV charts are simpler and signal faster than the SV charts.

1. Introduction

The Hotelling T^2 chart is the usual chart for detecting changes in the mean vector of multivariate processes. However, it is not always easy to convince practitioners accustomed to work with $\overline{X}$ values to consider a more complex statistic. The T^2 statistic is not only more complex in terms of computation but also with regard to its interpretation. If the statistical process control demands the monitoring of only two quality characteristics, the practitioner might prefer to work with two $\overline{X}$ charts, even knowing that the single T^2 chart was designed to control more than one quality characteristic. In comparison with the bivariate T^2 chart, the joint $\overline{X}$ charts have a better overall performance in signaling changes in the mean vector of correlated variables (Machado & Costa, 2008).

In a growing number of multivariate processes, the variables are cross-correlated and their observations are autocorrelated. Leoni, Machado, and Costa (2014) evaluated the effect of the cross-correlation and the autocorrelation on the performance of two combined $\overline{X}$ charts and on the performance of the Hotelling's T^2 chart. The overall conclusion is that the speed with which the charts signal reduces when the variable affected by the assignable cause is autocorrelated.

Leoni, Costa, and Machado (2015) obtained the cross covariance matrix of the rational sample mean vectors and investigated the joint effect of the correlation and autocorrelation on the T^2 chart's performance. Leoni, Costa, Franco, and Machado (2015) and Leoni, Machado, and Costa (in press), respectively, considered the skipping and the mixed sampling strategies to reduce the negative effect of the autocorrelation on the T^2 chart's performance. The skipping strategy was proposed by Costa and Castagliola (2011), and the mixed sampling strategy was proposed by Franco, Castagliola, Celano, and Costa (2013).

Wu and Spedding (2000) proposed to change the standard $\overline{X}$ chart's signaling rule of one point in the action region by the synthetic rule. Davis and Woodall (2002) obtained the steady-state properties of the $\overline{X}$ chart with the synthetic rule with the aim to prove its faster mean shift detection. The results of their studies motivated other researchers to consider the synthetic rule as an alternative to enhance the control charts' performance. A recent list includes the works of Haridy, Wu, Khoo, and Yu (2012), Calzada and Scariano (2013), Khoo, Wu, Castagliola, and Lee (2013), Haridy, Wu, Abhary, Castagliola, and Shamsuzzaman (2014), Chong, Khoo, and Castagliola (2014), Lee and Khoo (2014), Yeong, Khoo, Lee, and Rahim (2014), Guo, Wang, and Cheng (2015), Chew, Khoo, Teh, and Castagliola (2015), and Bajirao and Parasharam (2015). Machado and Costa (2014) proved

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that the side-sensitive version of the synthetic rule enhances the $\overline{X}$ chart's performance. Considering a wide range of mean shifts, the side sensitive feature reduces in 23%, on average, the time to detect the out-of-control condition. Costa and Machado (2015, 2016) considered the Markov chain approach to obtain the properties of the double sampling $\overline{X}$ charts and the variable sample size $\overline{X}$ charts, both with the synthetic and with the side sensitive synthetic rules. Celano and Castagliola (2016) investigated the synthetic rules and the gain in speed with which the charts, used to control the ratio of two normal variables, signal an out-of-control condition. Haq, Brown, and Moltchanova (2015, 2016) proposed new synthetic charts for monitoring process mean and dispersion. You, Khoo, Lee, and Castagliola (2015), Guo et al. (2015) and Yeong, Khoo, Yanjing, and Castagliola (2015) investigated the performance of several synthetic charts when the process parameters are estimated.

In this article, we propose the following synthetic charts to control bivariate processes with autocorrelated data: the T^2 chart and two simultaneous univariate $\overline{X}$ charts (the joint $\overline{X}$ charts). We consider the T^2 chart with the standard synthetic rule (Syn T^2 chart) and the $\overline{X}$ charts with two kinds of side-sensitive synthetic rules. The first one is side-sensitive with regard to the same variable (SV $\overline{X}$ charts) and the second one is side-sensitive with regard to both variables (BV $\overline{X}$ charts). When the $\overline{X}$ charts are in use, the synthetic rules depend on the values of the two sample means $(\overline{X}_1, \overline{X}_2)$. With the first side-sensitive rule (SV rule), the joint $\overline{X}$ charts signal in two cases: case (I) when the $\overline{X}_1$ and the $\overline{X}_2$ values of the same sample fall beyond their control limits; case (II) when the $\overline{X}_1$ or the $\overline{X}_2$ values of two different samples, not far from each other, fall beyond their control limits, except if the two points in the warning region are from the same variable and located on opposite sides of the center line. With the second side-sensitive rule (BV rule), the joint $\overline{X}$ charts signal in two cases: case (I) when the $\overline{X}_1$ and the $\overline{X}_2$ values of the same sample fall beyond their control limits; case (II) when the $\overline{X}_1$ or the $\overline{X}_2$ values of two different samples, not far from each other, fall beyond their upper (lower) control limits.

The paper is organized as follows: Section 2 is devoted to the presentation of the multivariate first order autoregressive model, VAR (1), and the bivariate cross-covariance matrix of the sample

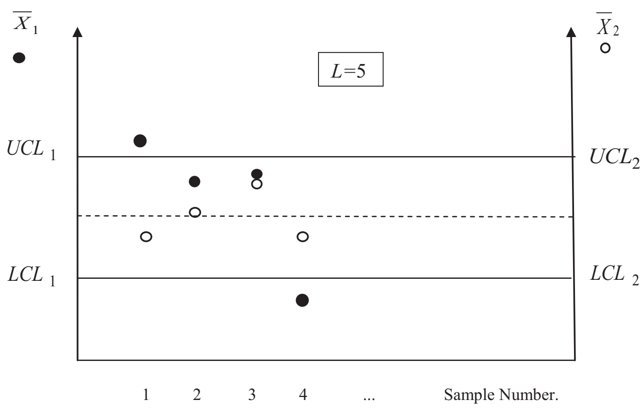

Fig. 1. Two points of the same variable in different warning regions.

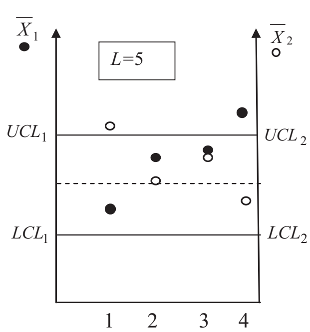

Fig. 2a. Two points in the same warning regions.

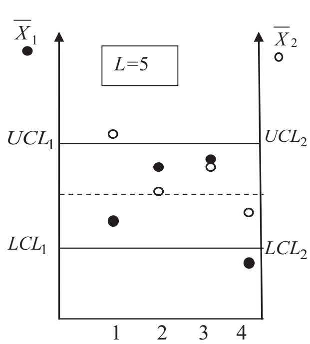

Fig. 2b. Two points in different warning regions.

Table 1
The $\overline{X}_1$ and $\overline{X}_2$ positions and the corresponding sample codes (SV $\overline{X}$ charts).

Sample codes	$\overline{X}_1$ position	$\overline{X}_2$ position
$\underline{2}$	$LCL_1 < \overline{X}_1 < UCL_1$	$\overline{X}_2 < LCL_2$
$\underline{1}$	$\overline{X}_1 < LCL_1$	$LCL_2 < \overline{X}_2 < UCL_2$
0	$LCL_1 < \overline{X}_1 < UCL_1$	$LCL_2 < \overline{X}_2 < UCL_2$
1	$\overline{X}_1 > UCL_1$	$LCL_2 < \overline{X}_2 < UCL_2$
2	$LCL_1 < \overline{X}_1 < UCL_1$	$\overline{X}_2 > UCL_2$

Table 2
The probabilities of the transition matrix (7).

Probabilities
$U_i = \Pr[\overline{X}_i > UCL_i, LCL_j < \overline{X}_j < UCL_j],\ i \neq j \in \{1,2\}$
$L_i = \Pr[\overline{X}_i < LCL_i, LCL_j < \overline{X}_j < UCL_j],\ i \neq j \in \{1,2\}$
$C = \Pr[LCL_1 < \overline{X}_1 < UCL_1, LCL_2 < \overline{X}_2 < UCL_2]$
$A = 1 - (C + U_1 + U_2 + L_1 + L_2)$
$B_i = 1 - (C + L_i),\ i \in \{1,2\}$
$D_i = 1 - (C + U_i),\ i \in \{1,2\}$

Table 3
The $\overline{X}_1$ and $\overline{X}_2$ positions and the corresponding sample codes (BV $\overline{X}$ charts).

Sample codes	$\overline{X}_1$ position	$\overline{X}_2$ position
$\underline{1}$	$LCL_1 < \overline{X}_1 < UCL_1$	$\overline{X}_2 < LCL_2$
	$\overline{X}_1 < LCL_1$	$LCL_2 < \overline{X}_2 < UCL_2$
0	$LCL_1 < \overline{X}_1 < UCL_1$	$LCL_2 < \overline{X}_2 < UCL_2$
1	$\overline{X}_1 > UCL_1$	$LCL_2 < \overline{X}_2 < UCL_2$
	$LCL_1 < \overline{X}_1 < UCL_1$	$\overline{X}_2 > UCL_2$

Table 4
The probabilities of the transition matrix (8).

Probabilities
$U = \sum_{i=1}^{2} \Pr[\overline{X}_i > LCL_i, LCL_j < \overline{X}_j < UCL_j],\ j \neq i \in \{1,2\}$
$L = \sum_{i=1}^{2} \Pr[\overline{X}_i < UCL_i, LCL_j < \overline{X}_j < UCL_j],\ j \neq i \in \{1,2\}$
$C = \Pr[LCL_1 < \overline{X}_1 < UCL_1, LCL_2 < \overline{X}_2 < UCL_2]$
$A = 1 - (C + U + L);\ B = 1 - (C + U);\ D = 1 - (C + L)$