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Reliability and sensitivity analysis of the controllable repair system with warm standbys and working breakdown

Tseng-Chang Yen [a], Wu-Lin Chen [b,*], Jia-Yu Chen [a]

[a] Department of Applied Mathematics, National Chung-Hsing University, Taichung, Taiwan, ROC
[b] Department of Computer Science and Information Management, Providence University, 200 Chung Chi Rd., Taichung 43301, Taiwan, ROC

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ABSTRACT

This paper deals with the reliability and sensitivity analysis of a controllable repair system with M operating units, S warm standby units, and an unreliable service station. Failure times and service times of operating units or standby units are assumed to follow exponential distributions. When the service station normally operates, it is subject to breakdowns; while a breakdown occurs, the service station requires repair at the repair facility. The times to breakdown and repair times of the service station are also assumed to be exponentially distributed. During a breakdown period of the service station, the service station is allowed to provide partial service with lower service rate. A practical justification of the system is also provided. This paper develops the expressions for system reliability, $R_Y(t)$, as well as the mean time to first failure, *MTTFF*. The sensitivity and relative sensitivity analyses of $R_Y(t)$ and *MTTFF* are performed. Finally, some numerical experiments are designed and performed to demonstrate the effects of system performance measures are affected by the change of each system parameter.

1. Introduction

In the repair system, it is often the case that the single service station will cost a lot when it is in normal operation. In this situation, that the single service station is always turned on seems an inappropriate design of the system. Hence, the controllable repair system was developed in the literature.

Many traditional queueing problems were considered under the assumption that the service station never fails. This assumption seems quite unrealistic; on the contrary, the service station may fail and then requires to be repaired in practice. Recently, some works studied the performance of the unreliable service station in which the service station is subject to non-deterministic breakdowns and then should be repaired. This paper considers the controllable repair system which consists of multiple operating units, warm standby units, and an unreliable service station.

The controllable repair system was first proposed by Yadin and Naor (1963) in 1963. Since then, several models regarding the control policy have been proposed; for example, see Heyman (1968), Tehgem (1986), Wang and Huang (1995) and Ke and Wang (2007).

On the other hand, the reliability issue of the queueing systems is quite critical on the design of these systems; therefore, the reliability analysis of different queueing systems has been continuing to be addressed in the literature. We only present some works which are more related to this study below. Ascher and Hung (1987) proposed a growth reliability model used to forecast future system reliability. Wang and Sivazlian (1989) derived the reliability of a repair system with multiple operating units, warm standbys and repairmen in 1989. In 1990, Vanderperre (1990) developed the reliability analysis of two standby units which have general probability distributions. Cao (1994) studied the reliability and sensitivity analysis of an M/G/1 queueing system in 1994. Tang (1997) proposed the system availability of an M/G/1 queueing system where the server was subject to breakdowns in 1997. Ke, Lee, and Wang (2007) investigated the reliability and sensitivity analysis of a repairable system with multiple unreliable service stations and standby switching failures in 2007. In 2009, Wang and Chen (2009) compared the system availability of three systems with general repair times, reboot delays and switching failures. Yuan and Meng (2011) addressed reliability analysis of a warm standby repairable system. In 2014, reliability analysis of a single warm- standby system subject to repairable and nonrepairable failures was proposed by Wells (2014). Kuo, Sheu, Ke, and Zhang (2014) presented the reliability analysis for a retrial system with mixed standbys. Levitin, Xing, and Dai (2015) analyzed the reliability of non-coherent warm standby systems with reworking. Based on the above descriptions, there are many studies to address the

* Corresponding author.
E-mail addresses: tcyen@amath.nchu.edu.tw (T.-C. Yen), wlchen@pu.edu.tw (W.-L. Chen), d9853011@mail.nchu.edu.tw (J.-Y. Chen).

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reliability analysis of queueing systems, most of which discussed how to effectively utilize the standby(s) in order to enhance the system reliability. However, the reliability analysis of the machine repair system with the controllable and unreliable server is not addressed in the literature.

The studies related working breakdowns are described as follows. In 2012, Kalidass and Kasturi (2012) considered an M/M/1 classical queueing model with working breakdowns. Here, the working breakdowns mean that the server can also provide partial service with lower service rate during its breakdown periods. Li, Wang, and Zhang (2013) investigated the system with working breakdowns and balking tasks.

In addition to the reliability analysis, the optimal analyses of the machine repair systems with different settings were also proposed in the literature. For example, see Ke, Hsu, Liu, and Zhang (2013), Hsu, Ke, Liu, and Wu (2014), Wu and Ke (2014) and Ke, Liu, and Wu (2015). Sensitivity analysis and relative sensitivity analysis of the reliability for a repairable system with imperfect coverage under service pressure condition was proposed by Wang, Yen, and Jian (2013). Recently, Chen, Wen, and Chen (2016) investigated the reliability of the machine repair system with an unreliable and multiple-vacation server.

As far as we know, the reliability analysis of the controllable repair system with the unreliable service station has never been discussed in the literature. The main objectives of this study can be listed below:

(1). Derive the expressions for both the system reliability and the mean time to first failure by using the Laplace transform technique.
(2). Perform the sensitivity and relative sensitivity analyses of the system reliability and the mean time to first failure.
(3). Design and perform some numerical experiments.

2. Problem description and notations

This paper considers a repairable system with M identical units operating in parallel, S warm standby units, and an unreliable service station. Let us assume that failed units arriving at the service station form a single waiting line and are served in the order of their arrivals; i.e., according to the first-come, first-serve discipline. Failure times of both operating and standby units are assumed to have exponential distributions with rates λ and $\eta(0 < \eta < \lambda)$, respectively. If an operating unit fails, it is replaced by a standby unit right at once if any is available. It is assumed that the switch is perfect (always successful) and the switchover time is instantaneous. When a standby unit becomes an operating one, its failure characteristic will be that of an operating unit. The control policy of the system is that the service station is turned on when there are N or more failed units in the system; it will remain on until there is no failure unit (and then the service station is turned off). While the service station is available, it is also subject to breakdown and its breakdown times and repair times are assumed to follow exponential distributions with breakdown rate α and repair rate β. When the service station is available and working, the repair times of failed units are also assumed to have an exponential distribution with rate μ_1. It is also assumed that when the service station is on and breakdown, it can also provide partial service with lower service rate $\mu_2(0 < \mu_2 < \mu_1)$. Suppose that the service station can serve only one failed unit at a time and the service time of a unit is independent of the arrival time. Once a unit is repaired, it is as good as new.

System reliability is analyzed based on the assumption that the system fails as soon as all the units fail. Hence, if let n denote the number of failed units in the system, the system fails if and only if $n = M + S$.

The following notations are used in the following sections to analyze the system:

M	number of operating units
S	number of warm standby units
L	number of total units $(L = M + S)$
N	control policy threshold
n	number of failed units in the system
λ	failure rate of an operating unit
η	failure rate of a warm standby unit
μ_1	service rate of the service station under its working condition
μ_2	service rate of the service station under its breakdown condition
α	breakdown rate of the service station
β	repair rate of the service station
λ_n	failure rate when there are n failed units in the system
$p_i(n;t)$	probability that there are n failed units in the system at time t when the system is in state i, where $i = 0$ denotes the service station is turned off, $i = 1$ denotes the service station is turned on and working, and $i = 2$ denotes the service station is turned on but breakdown
$p_i^*(n;s)$	Laplace transform of $p_i(n;t)$
$P_i^*(s)$	column vector of $p_i^*(n;s)$
$P_i(0)$	column vector of $p_i(n;0)$
$R_Y(t)$	system reliability
MTTFF	mean time to first system failure

Before analyzing the system reliability and *MTTFF*, we provide a practical justification of the system considered in this paper as follows. Please see Armbrust et al. (2010) and Pegasus (2013).

2.1. Practical justification of the System

A virtual machine (VM) is the software implementation installed in a virtual storage, called a root file-system, to emulate a physical computing environment, and therefore VMs are very suitable for system management practice and for application development and test. Moreover, virtualization is the core technology of cloud computing. For illustrative purpose, we consider a private cloud data center which provides Platform as a Service (PaaS) (Armbrust et al., 2010) via virtual machines (treated as both operating units and standby units) and is managed by Pegasus (2013) open source PaaS manager which deploys and monitors VMs. To guarantee the high availability of service, a breakdown operating VM is instantly replaced by a standby VM if available, and the breakdown VMs are sent to the VM repair server (treated as the service station) to check and repair the broken VM. Fig. 1 shows the configuration of the private cloud data center.

Initially, all the M operating VMs and S warm standby VMs are working, and the VM repair server is turned off to reduce the power consumption. The operating VM fails independently of the state of the warm standby VM and vice versa. Let us assume that failed VMs are repaired in the order of their arrivals; i.e., according to the first-come and first-serve discipline. Let the time-to-failure of the operating VM and the time-to-failure of the warm standby VM be exponentially distributed with parameters λ and η $(0 < \eta < \lambda)$, respectively. If an operating VM fails, it is replaced by a standby VM immediately if any is available. The *Pegasus* manager turns on the VM repair server when N $(N \geqslant 1)$ or more failed VMs are in the system, and this server is turned off only when all failed VMs are repaired. During the repair period, the VM repair server is subject to the process error, and therefore the VM repair server is in the failure state in this situation. The failure times and repair times of the VM repair server are assumed to be exponential distributions with failure rate α and repair rate β. When the VM repair server is fully functional, the repair times of failed VMs are also assumed to have exponential distribution with rate