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Integer programming techniques for makespan minimizing workforce assignment models that recognize human learning

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ABSTRACT

We study optimization techniques for makespan minimizing workforce assignment problems wherein human learning is explicitly modeled. The key challenge in solving these problems is that the learning functions that map experience to worker productivity are usually nonlinear. This paper presents a set of techniques that enable the solution of much larger instances of such problems than seen in the literature to date. The first technique is an exact linear reformulation for the general makespan minimizing workforce assignment models with learning. Next, we introduce a computationally efficient means for generating an initial feasible solution (which our computational experiments indicate is often near-optimal). Finally, we present methods for strengthening the formulation with cover inequalities and a lower bound on the objective function value of the optimal solution. With an extensive computational study we demonstrate the value of these techniques and that large instances can be solved much faster than have previously been solved in the literature. To focus the paper on the presented methodology, we solve a makespan minimizing workforce assignment problem that has few complicating constraints. However, the techniques can be adapted to speed up the solution of most any makespan minimizing workforce assignment problem.

1. Introduction

Workforce planning is the process by which companies deploy their workforce to complete the tasks of the organization. In making these assignments, it is important to recognize that the assignments that they make in the short term have a longer term impact on the organization's capacity. In particular, workers learn and become more productive the more experience that they gain.

In this research, we assume a set of heterogeneous workers and a set of unrelated jobs that must be completed over the course of a given horizon. As workers work on a particular job, they become more experienced and thus more productive. The increase in productivity is often referred to as "learning." We assume that each individual worker has a known learning curve that determines that worker's productivity. Importantly, we account for the fact that some workers learn faster than others. The objective is to minimize the time required to complete all of the jobs (known as minimizing the makespan). We call this problem the makespan minimizing workforce assignment problem with learning.

The key challenge in solving the makespan minimizing workforce assignment problem with learning is that the learning curves are usually nonlinear. As a result, most work in the literature is limited to solving small-sized problems. This paper presents a set of techniques that enable the solution of much larger instances of such problems. In particular, this paper presents three techniques that are contributions to the literature on solving makespan minimizing workforce assignment problems that explicitly model learning. First, we present an exact linear reformulation for the general makespan minimizing workforce assignment model with learning. While the reformulation technique is adapted from the literature, this paper is the first to apply it to the learning function considered in this paper and in the context of a makespan minimizing workforce assignment problem. Then, we introduce a computationally efficient means for generating an initial feasible solution (which our computational experiments indicate is often near-optimal). We also present methods for strengthening the formulation with cover inequalities and a lower bound on the objective function value of the optimal solution. With an extensive computational study we demonstrate the value of these techniques. To focus the paper on the techniques we present we solve a makespan minimizing workforce assignment problem that has few complicating constraints. However, the techniques we present

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can be adapted to speed up the solution of most any makespan minimizing workforce assignment problem. Further, the presented techniques do not depend on the particular learning and forgetting function.

This paper is organized as follows. Section 2 presents the literature review on workforce planning and learning curves and introduces more precisely the problem description. Section 3 first presents a nonlinear formulation of the makespan minimizing workforce assignment problem with learning. The section then introduces the linear reformulation of the problem, a way to generate initial feasible solutions, as well as the inequalities and bound that strengthen the formulation. Section 4 presents our datasets and computational experiments demonstrating the value of the reformulation, the initial solution, cover inequalities, and lower bound. Section 5 offers the conclusions and discusses future avenues of research.

2. Literature review

In this paper, we focus on scheduling jobs to workers with the goal of minimizing the makespan. Our models account for the fact that workers learn and thus increase productivity as they gain experience.

The most closely related work is that of Corominas, Olivella, and Pastor (2010) who introduce a piecewise linearization of a learning function for a task assignment model. The largest problem solved has five tasks and four workers and they do so with a two-segment piecewise linear approximation to linearize a concave learning function. Olivella, Corominas, and Pastor (2013) extend Corominas et al. (2010) to consider due dates and cross-training goals and use a similar solution approach to Corominas et al. (2010). The paper solves problems with up to four workers and eight tasks. Heimerl and Kolisch (2010) addresses a problem similar to Olivella et al. (2013) while also including forgetting and company skill levels. Using a nonlinear programming solver that cannot guarantee optimal solutions, Heimerl and Kolisch (2010) solve problems with six workers and 20 tasks. In contrast, in the work presented in this paper, we solve exactly problems with up to 20 workers and 30 tasks. A comprehensive review of other work that incorporates learning into workforce planning models can be found in Hewitt, Chacosky, Grasman, and Thomas (2015a, 2015b).

Using a model for assembly-line production first presented in Nembhard and Norman (2007, chap. 4), Hewitt et al. (2015a, 2015b) introduces a technique for deriving exact linear reformulations of nonlinear learning functions. The reformulation technique presented in Hewitt et al. (2015a, 2015b) models nonlinear functions with discrete domains and ranges as sets of binary and linear variables and constraints. Hewitt et al. (2015a, 2015b) can solve problems up to 20 workers and 40 tasks in less than an hour. However, Hewitt et al. (2015a, 2015b) considers a different non-linear model of human learning than we consider in this paper. Thus, while this paper adapts the reformulation technique presented in Hewitt et al. (2015a, 2015b), it differs in three important ways: (1) We consider a different class of scheduling problems (makespan minimizing workforce assignment problems), (2) we consider a different quantitative model of how experience translates to proficiency, and (3) we provide techniques that significantly reduce the solve time of the reformulated model.

Table 1 summarizes the literature most closely related to that in this paper. The first column cites the paper being summarized and the second notes from what paper the summarized paper's model is derived. The third and fourth columns identify whether or not the model includes cross-training goals and whether or not the model is nonlinear, respectively. The fifth and sixth columns (found in the second layer of the table) highlight the solution method and whether or not the method is a heuristic method, respectively, The final three columns indicated the largest problem solved in each paper by indicating the number of workers, tasks and time periods, respectively, of the largest problem solved.

Table 1
Summary of related literature.

Publication	Related models	Cross-training goals	Nonlinear model
Nembhard and Norman (2007, chap. 4)			
Corominas et al. (2010)	–		✓
Heimerl and Kolisch (2010)	–	✓	✓
Olivella et al. (2013)	Corominas et al. (2010)	✓	✓
Hewitt et al. (2015a, 2015b)	Nembhard and Norman (2007, chap. 4)		
This paper	Corominas et al. (2010)		

Publication	Solution approach	Heuristic	Workers	Tasks	Periods
Nembhard and Norman (2007, chap. 4)	Nonlinear programming		2	4	10
Corominas et al. (2010)	Piecewise linearization	✓	4	5	20
Heimerl and Kolisch (2010)	Primal–dual interior point	✓	6	4	5
Olivella et al. (2013)	Approximate convex piecewise linearization	✓	4	8	40
Hewitt et al. (2015a, 2015b)	Reformulated mixed integer program		20	40	40
This paper	Reformulated mixed integer program		20	30	NA

The study of how humans learn has a long history. With the introduction of his "power model," Wright (1936) is often credited with introducing the first mathematical description of the relationship between experience and productivity. These mathematical descriptions are often called learning curves. To the best of the authors' knowledge, all of the commonly employed learning curves (power, exponential, hyperbolic) are nonlinear. Anzanello and Fogliatto (2011), Dar-El (2000), Jaber (2006, chap. 30), Jaber and Sikström (2004) provide broad and thorough surveys of the subject. Hewitt et al. (2015a, 2015b) provide a review of learning curves in optimization models.

We apply the three-parameter exponential learning curve model first proposed by Bevis, Finniear, and Towill (1970). The exact description of the model is given in the next section. However, the methods discussed subsequently in this paper do not require a particular learning and forgetting function, only that experience be measured in discrete units. Examples of other such learning functions include Gutjahr, Katzensteiner, Reiter, Stummer, and Denk (2008), Heimerl and Kolisch (2010), Nembhard and Norman (2007, chap. 4), and Sayin and Karabati (2007).