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An effective placement method for the single container loading problem

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ABSTRACT

This study investigates a three-dimensional single container loading problem, which aims to pack a given set of unequal-size rectangular boxes into a single container such that the length of the occupied space in the container is minimized. Motivated by the practical logistics instances in literature, the problem under study is formulated as a zero-one mixed integer linear programming model. Due to the NP-hardness of the studied problem, a simple but effective loading placement heuristic is proposed for solving large-size instances. The experimental results demonstrate that the developed heuristic is capable of solving the instances with more than two hundred boxes and more efficient than the state-of-the-art mixed integer linear program and existing heuristic methods.

1. Introduction

The container loading problem (CLP), which is also referred to as the packing problem, plays a crucial role in logistics planning and scheduling, and widely arises in various real-world applications, such as electronic, steel, paper, textiles, manufacturing and transportation industries. In this paper, we consider the three-dimensional container loading problem (3DCLP), where a given set of nonuniform-size rectangular boxes is to be packed within an enveloping rectangular container such that the length of the occupied space in the container is minimized. All the boxes are assumed to be orthogonally positioned, i.e. each edge of the box is parallel to one axis of the container. The objective function addressed in this study is different from the criterion of minimizing the volume of the occupied space in that an occupied space with the minimum length unnecessarily retains the minimum volume. For example, a container with a length of 6-m, a width of 3-m, and a height of 2-m is given for packing six cube boxes with a 1-m side length and one rectangular box with a 1-m length, 1-m width, and 0.5-m height. If minimizing the volume of the occupied space is considered, an optimal solution with the length equaling 3.5 m and the volume equaling $(3.5 \times 2 \times 1) = 7\ \mathrm{m}^3$ is shown in Fig. 1(a). If the criterion is the minimization of the length, an optimal solution with the length equaling 1.5 m and the volume equaling $(1.5 \times 3 \times 2) = 9\ \mathrm{m}^3$ is depicted in Fig. 1(b). Motivated by the practical instances in logistics (cf. Chen, Lee, & Shen, 1995; Hu, Li, & Tsai, 2012), the studied problem could arise from the container loading for a truck, which delivers goods or parcels of various types to several different shops or transshipment points along a route. To process the unloading efficiently, the container is divided into several sections, each of which is stacked with the goods ordered by an individual shop or the parcels to be transferred to a transshipment station. As shown in Fig. 2, minimizing the length of the section required to pack goods for each shop yields the maximal number of shops or transshipment points to which a truck can make deliveries with a single trip, and thus reduces inventory and transportation costs. For each shop or transshipment station, i.e. each section in the container, the 3DCLP under study is considered. Furthermore, the generalized assumption that the sizes of rectangular boxes are various and even customer-defined is not uncommon and actually practical in Taiwan's logistics industries, such as Hsin-Chu Transportation Logistics, Kerry TJ Logistics, Taiwan Pelican Express, T-cat Takkyubin, and Taiwan Post (cf. Chou & Lu, 2009).

In the past decades, the solution approaches proposed to cope with variants of CLPs can be divided into two classes: heuristic algorithms and deterministic methods. Owing to the great complexity of the CLPs in nature, most research has focused on the development of heuristic methods. Considering the two-dimensional pallet loading problem, Hodgson (1982) developed an integrated approach of dynamic programming and heuristic procedure to minimize the number of pallets for loading a given set of boxes. George and Robinson (1980) investigated a problem of packing a given set of boxes into a container with a fixed volume

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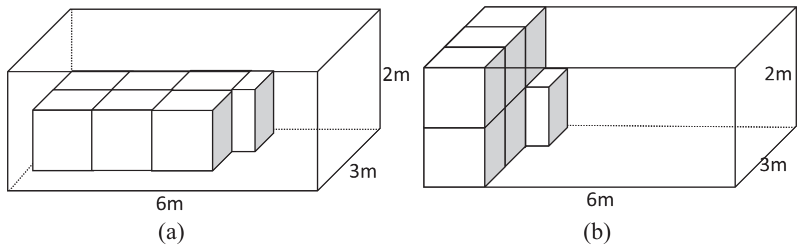

Fig. 1. Optimal solutions for the volume minimization (a) and the length minimization (b).

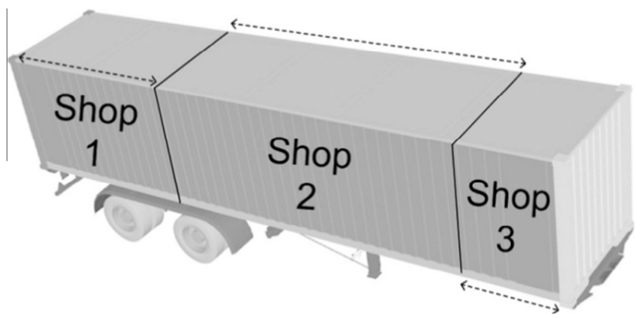

Fig. 2. Three container sections for three individual shops.

and proposed a heuristic for the sequencing and positioning such that all the boxes can be fitted in. Bischoff and Marriott (1990) presented a comparative analysis of fourteen heuristic rules for the CLP and examined the impact of the number of different item types in a load on the loading efficiency. Considering the weight and the load bearing abilities of the boxes in the CLP, Ratcliff and Bischoff (1998) proposed an approach to adapt the solution procedures in the literature to deal with fragility consideration. Faina (2000) developed a geometrical model that reduces the CLP to a finite enumeration scheme, and presented a statistical optimization algorithm based on simulated annealing. Eley (2002) designed a greedy heuristic coupled with a tree search procedure for the single-container and multiple-container packing problems. Pisinger (2002) presented a heuristic algorithm to decompose the CLP into a number of layers, which are divided into a number of strips. Then the decomposed sub-problem of packing strips can be formulated as the well-known knapsack problem. Investigating the arbitrary m-dimensional CLP, Lins, Lins, and Morabito (2002) proposed a recursive uniform algorithm to partition the container into various sub-containers, each of which can be recursively partitioned into further sub-containers. A parallel tabu search algorithm based on the multi-search threads for the CLP was presented by Bortfeldt, Gehring, and Mack (2003). Then Hifi (2004) developed an integrated algorithm of dynamic programming and graph search procedure with depth-first search strategy for solving the weighted three-dimensional cutting problem, which is a variant of CLP. Considering the three-dimensional strip packing problem, Bortfeldt and Mack (2007) designed a heuristic algorithm which was derived from the layer-building approach proposed by Pisinger (2002). Almeida and Figueiredo (2010) presented an alternative non-linear formulation containing additional restrictions on the placement and designed a heuristic algorithm for the CLP. Fanslau and Bortfeldt (2010) presented a tree search algorithm for the 3DCLP, where a special form of tree search keeps the search effort low and ensures a suitable balance between search diversity and foresight during the search. Zhang, Peng, and Leung (2012) proposed a block-loading algorithm based on multi-layer search for the 3DCLP with depth-first and multi-layer search for determining the selected bock in each packing phase. The computational results in Zhang et al. (2012) showed that their developed algorithm outperforms the method presented by Fanslau and Bortfeldt (2010). Zhu, Huang, and Lim (2012) addressed the cost minimization in the multiple-container packing problem formulated as a set cover problem, and presented a prototype column generation scheme.

In the relevant literature on deterministic methods, the CLPs are generally modeled as a mathematical program. Chen et al. (1995) considered the 3DCLP, which aims to select a number of containers to pack a given set of boxes. They proposed a mixed integer linear programming (MILP) formulation and solved the small-size problems by the LINGO package. Then Tsai and Li (2006) first adapted the MILP model proposed by Chen et al. (1995) to present a mixed integer nonlinear programming (MINLP) model for the 3DCLP which aims to pack a given set of boxes into a container with the minimum volume. They modified the MINLP by reducing the number of binary variables and utilizing a piecewise linearization technique to find a global optimum within a tolerable error. Tsai, Wang, and Lin (2014), subsequently, converted the original MINLP into an MILP with an improved piecewise linearization technique using fewer extra zero-one variables and constraints to enhance the computational efficiency. On the other hand, Hu et al. (2012) developed a transformation method to convert the nonlinear objective function in the 3DCLP into an increasing function with single variable and two fixed parameters. A transformed MILP was then decomposed into several sub-problems, which were solved in parallel by a proposed distributed genetic algorithm.

The NP-hardness of the studied 3DCLP, which is a generalization of the bin-packing problem, indicates that it is very unlikely to devise a polynomial-time exact algorithm for yielding an optimal solution to the 3DCLP unless P = NP. To cope with the medium- or large-scale 3DCLP, developing an effective heuristic is relatively applicable. We observe that most existing heuristics for the 3DCLP are based on the wall-building approach, which suffers from the following deficiencies in efficiency and effectiveness: (i) A backtracking probability is necessary for solution evolution, which could incur a considerable computation time; (ii) The generated solution would have numerous unutilized separated spaces and thus a low container space utilization. In this paper, we proposed a simple but effective loading placement heuristic for the studied 3DCLP. By comparison with the state-of-the-art deterministic MILP model and existing heuristic techniques, the developed heuristic algorithm is capable of generating quality solutions to the benchmark dataset efficiently.

The rest of the paper is organized as follows. In Section 2, a reference MILP of the 3DCLP is introduced. A loading placement heuristic algorithm is proposed in Section 3. Numerical experiments are conducted in Section 4. Finally, some concluding remarks are made in Section 5.

2. Reference MILP model

Assume that n nonuniform-size rectangular boxes, each of which has a specific length, width and height, are given. Denote