Computers & Industrial Engineering 95 (2016) 83–96

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

A slack based network DEA model for generalized structures: An axiomatic approach

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ARTICLE INFO

Article history:
Received 20 February 2015
Received in revised form 16 December 2015
Accepted 21 February 2016
Available online 2 March 2016

Keywords:
Data envelopment analysis
Network DEA
Overall and divisional efficiency
Efficient targets
Dual pairs

ABSTRACT

Recently, network data envelopment analysis (Network DEA) models have been developed in order to assess decision making units (DMUs) regarding their internal structures. Each DMU consists of a network of related divisions, and each division takes over some part of the production process. Different models are represented in multiplier forms and envelopment forms, which each of them considers some aspects of assessments (such as preference weights of inputs/outputs and divisional efficiencies or efficient targets). However, these models are not necessarily dual equivalent forms as well as traditional DEA models. Also as will be shown in this paper, in some envelopment form models, the proposed efficiency targets are not practically applicable in some network structures. In this paper we offer a model using an axiomatic approach for a general network structure which does not have the pitfalls of efficient targets. Also an interpretation of its dual multiplier form is represented.

1. Introduction

Data envelopment analysis (DEA) was first introduced by Charnes, Cooper, and Rhodes (1978) as an efficiency measurement tool for similar decision making units (DMUs). Now, DEA models are widely used in practice and have also been widely improved theoretically (see e.g. Cook & Seiford, 2009). In conventional DEA models, DMUs are considered as black boxes. Some inputs (i.e. the main inputs) are externally devoted to the DMUs, and the DMUs export some final outputs (i.e. the main outputs) to the outside of the DMU (Charnes et al., 1978; Banker, Charnes, & Cooper, 1984). But in network DEA models, a DMU is considered to be a network of related divisions. Each division takes over a part of the production process (Castelli, Pesenti, & Ukovich, 2010). Some divisions are dependent on intermediate productions (i.e. intermediate factors or intermediate measures), which are produced in some other divisions. In fact, some divisions receive some main inputs and transform them into intermediate factors before they are finally changed into the main outputs in the production sequence among divisions.

Network DEA was established by Färe and Grosskopf (1996) and then developed by Färe and Grosskopf (2000). Most of the existing network DEA models relate to special types of network structures which are particularly applicable such as two-stage structures (Liang, Cook, & Zhu, 2008; Chen, Cook, Li, & Zhu, 2009a; Chen, Liang, & Zhu, 2009b; Chen, Cook, & Zhu, 2010; Cook, Liang, & Zhu, 2010a; Li, Chen, Liang, & Xie, 2012), multi-stage structures (Golany, Hackman, & Passy, 2006), and series and parallel structures (Kao, 2008). However, in recent years, some models have been produced that represent general network structure[1] (i.e. general network DEA models). One of the fundamental general network DEA models, was introduced by Tone and Tsutsui (2009). They represented overall efficiency measures and also divisional efficiency measures in an envelopment form model. Their definitions were reviewed by Fukuyama and Mirdehghan (2012). They offered examples with regard to which the divisional efficiency and also the overall efficiency as defined by Tone and Tsutsui, did not work for their examples. After that, Lozano (2011) presented an axiomatic approach for his envelopment form model, and formulated the production possibility set. Later, he also extended a SBM network DEA model based on this production possibility set (Lozano, 2015). In multiplier form, Kao (2009) also represented an approach which is

[1] In a general network structure we have a finite number of related divisions. Each division may receive its inputs from other divisions or from out of the DMU, and produce outputs for other divisions or as final products. But in a specified period of time, since a division cannot use its productions as an input (either directly or indirectly), we do not have loops in a general network structures. Looped structures are studied in dynamic models which we do not deal with in this paper.

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http://dx.doi.org/10.1016/j.cie.2016.02.017
0360-8352/

based on the idea that a production factor must have a unique weight in each division it deals with. In continue, Kao, Chan, and Wu (2014) have developed a multi objective program in multiplier form too. Both in multiplier and envelopment form models, some researchers attempted to find a direct relation among overall efficiency and divisional efficiencies. For a general network structure, Kao (2009, 2014) and also Cook, Zhu, Yang, and Bi (2010b) decomposed overall efficiency into divisional efficiencies based on multiplier form models. Also Kao (2009) represented another efficiency decomposition in envelopment form.

In another paper, Chen, Cook, Kao, and Zhu (2013) investigated the pitfalls in network DEA models. In some network DEA models, efficiency measures are not truly defined. For example Tone and Tsutsui (2009) did not consider intermediate factors in undertaking divisional efficiency measures. Moreover, based on their definitions of overall and divisional efficiencies, the improvement of the overall efficiency was equal to the improvement in all divisional efficiencies. This viewpoint was also reviewed by Chen et al. (2013), because if we change the amount of an intermediate factor, then the divisional efficiency increases for one of its dependent divisions and decreases for the other dependent divisions. More precisely, if we increase (decrease) the amount of an intermediate factor, the divisional efficiency increases (decreases) for its producer division, and decreases (increases) for its consumer division. In this paper we ultimately want to decrease the main inputs of the DMU and increase its main outputs. To achieve this, the intermediate factors are free to be increased or decreased, and consequently the divisional efficiency of divisions may increase or decrease. In other words our target is to improve the overall efficiency with possible sacrifice of divisional efficiency of some divisions. So the improvement of overall efficiency is not equal to the improvement of all divisional efficiencies.

In previous studies, the efficiency targets were investigated in a special type of network structures (Chen et al., 2009b). But in general network DEA models yet represented in envelopment forms, the proposed efficiency targets have some pitfalls. Since Tone and Tsutsui's model (2009) does not give true efficiency scores in some network structures, the obtained efficiency targets cannot be used for overall efficiency improvement. Also in this paper, we will see that the efficiency targets represented by Lozano's model (2015) cannot be applicable in some network structures, because this model do not consider the distribution of intermediate factors' flows among divisions (see Section 5.2). In this paper an axiomatic approach is represented in which the resultant model gives the efficient flows in all divisions. It determines the optimal distribution of intermediate factors in order to increase the main outputs and also decrease the main inputs of DMUs (i.e. the overall efficiencies of DMUs are increased).

Besides the above-mentioned pitfalls, the existing network DEA models are not necessarily equivalent dual forms. The multiplier forms and envelopment forms give different results. So a unified interpretation does not exist for envelopment and multiplier forms as well as for conventional DEA models. In this paper we also investigate the dual (multiplier) form of the proposed model, and offer an interpretation for that which can be enumerated as a generalization of interpretations in the multiplier form of conventional DEA models. So we would have dual pair of network DEA models with unified interpretations.

The structure of the paper is as follows: Section 2 represents some basics and introduces the notations used in this paper. In Section 3, a more comprehensive review on the existent general network DEA models is represented (esp. Kao, 2009; Tone & Tsutsui, 2009; Lozano, 2015). In Section 4, we offered our considered model, based on an axiomatic approach in a general network structure. Also an interpretation is presented for its dual multiplier form. In Section 5, the proposed model introduced in Section 4, is compared with the other general network DEA models discussed in Section 3. In Section 6, an illustrative application is investigated in order to clarify the properties of the proposed model and a comparison is made with the models discussed in Section 3. Finally, in Section 7 we conclude by offering an overall discussion and future studies.

2. Basics and definitions

Assume that we have J number of DMUs ($j = 1, \ldots, J$). Each DMU consists of K divisions ($k = 1, \ldots, K$) and Q types of factors exist as input and output factors (either main or intermediate factors). Suppose that $k \underset{q}{\rightarrow} h$ denotes the flow of factor q from division k to division h. Now consider that N_k is the set of main inputs used by division k and M_k is the set of main outputs produced by division k. Also $\widetilde{N}_k$ and $\widetilde{M}_k$ are the sets of intermediate input factors and intermediate output factors for division k respectively. So:

$$\begin{array}{ll} \widetilde{M}_k = \{q \mid \exists h : k \underset{q}{\rightarrow} h\} & (a) \\ \widetilde{N}_k = \{q \mid \exists h : h \underset{q}{\rightarrow} k\} & (b) \\ M_k = \{q \mid q \text{ is main output for div. } k\} & (c) \\ N_k = \{q \mid q \text{ is main input for div. } k\}. & (d) \end{array} \tag{1}$$

In this paper we want to apply our proposed model for any arbitrary network structure of a DMU (i.e. general network structure). Consequently, we should note a number of points. First of all, we should note that some of these sets may be empty for some divisions. For example, M_k is empty iff division k produce only intermediate factors for other divisions, and do not produce any main output (see divisions 1 or 2 or 3 of Fig. 1). Secondly, we should note that, for a considered division k, some of these sets may have common members. For example, in a specified division k we may have a type of output in terms of which a portion of this output is used internally in other divisions of the DMU, and the rest of this output is exported to the outside of the DMU as a main output. So this factor is a common member in M_k and $\widetilde{M}_k$ for this division k (see q_4 in division #5 of Fig. 1). Similarly, factor q_3 is simultaneously a main input and an intermediate input for division #3 in Fig. 1.

Although some of these sets may have common members, the set of inputs and outputs for a fixed division k are distinct. ($\forall k,\ (M_k \cup \widetilde{M}_k) \cap (N_k \cup \widetilde{N}_k) = \varphi$.)

The set of main inputs (main outputs) for the DMU, is the set of main inputs (main outputs) used (produced) by all of its divisions. Let's name these sets as $N = \bigcup_k N_k$ and $M = \bigcup_k M_k$ respectively. For a $q \in N$ (or $q \in M$), if it belongs to two or more N_k's (or M_k's) this means that this main input (or output) is shared among several divisions. In this case, we have a network structure with shared resources (or products).

The amount of flow of an intermediate factor q from division k to division h in DMU$_j$ is denoted by $z_{qj}^{(k,h)}$. Also the amount of main input q devoted to division k and the amount of main output q produced by division k are denoted by x_{qj}^k and y_{qj}^k respectively.

3. Literature review on general network DEA models

In this section, we concentrate on some network DEA models in the literature, which are represented in the form of general network structures (i.e. general network DEA models). In order to compare these with the model proposed in the next sections,