Computers & Industrial Engineering 99 (2016) 63–77

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

# Multiple attribute group decision making based on IVHFPBMs and a new ranking method for interval-valued hesitant fuzzy information

Yingdong He, Zhen He, Liangxing Shi *, Shanshan Meng

College of Management and Economics, Tianjin University, Tianjin, China

ARTICLE INFO

Article history:
Received 17 March 2015
Received in revised form 5 June 2016
Accepted 3 July 2016
Available online 6 July 2016

Keywords:
Interval-valued hesitant fuzzy set
2nd-order dispersive central polymerization degree function
Power Bonferroni mean
Multiple attribute group decision making

ABSTRACT

The interval-valued hesitant fuzzy set is a significant tool to express the uncertain information. In this paper, we define the interval-valued hesitant fuzzy 2nd-order central polymerization degree ($IVHFCP_2$) function and the interval-valued hesitant fuzzy 2nd-order dispersive central polymerization degree ($IVHFDCP_2$) function to further compare different interval-valued hesitant fuzzy sets. To capture much more information for the multiple attribute group decision making, we combine the Bonferroni mean with the power average operator to accommodate to interval-valued hesitant fuzzy environments and develop the interval-valued hesitant fuzzy power Bonferroni mean (*IVHFPBM*) and the interval-valued hesitant fuzzy weighted power Bonferroni mean (*IVHFWPBM*). We investigate the desirable properties of the new interval-valued hesitant fuzzy aggregation operators and discuss their special cases in detail. Finally, the new aggregation operators are applied to interval-valued hesitant fuzzy multiple attribute group decision making and a numerical example is given to illustrate the effectiveness of the presented approaches.



## 1. Introduction

Atanassov (1986) introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of a fuzzy set (Zadeh, 1965). Each element in the IFS is expressed by an ordered pair, and each ordered pair is characterized by a membership degree and a non-membership degree. When discussing the membership degree of $x$ in $A$, different decision makers may assign different values, for example, one wants to assign 0.3 while another wants to assign 0.5. But they are not willing to compromise with each other, for which, Torra (2010) and Torra and Narukawa (2009) extended fuzzy sets (Zadeh, 1965) to hesitant fuzzy sets (*HFS*s), and the above membership of $x$ in $A$ can be presented as $\{0.3, 0.5\}$. Torra and Narukawa (2009) further discussed the similarities between *HFS*s and intuitionistic fuzzy sets (*IFS*s) (Atanassov, 1986), and showed that the envelope of a hesitant fuzzy set is an intuitionistic fuzzy set. Rodriguez, Martinez, and Hsrrera (2012) combined linguistic sets with hesitant fuzzy sets and proposed hesitant fuzzy linguistic term sets. Wei (2015) combined uncertain linguistic sets with interval-valued hesitant fuzzy sets and proposed interval-valued hesitant fuzzy uncertain linguistic sets.

Other important generalizations of fuzzy sets and their applications can refer to the research on fuzzy graphs (Akram, 2011), linguistic fuzzy sets (Merigó & Gil-Lafuente, 2009, 2013; Merigó, Gil-Lafuente, Zhou, & Chen, 2012; Zadeh, 1975), type-2 fuzzy sets (Castillo & Melin, 2012; Fazel Zarandi, Gamasaee, & Turksen, 2012; Galluzzo & Cosenza, 2011; Greenfield, Chiclana, John, & Coupland, 2012; Zhai & Mendel, 2011), intuitionistic fuzzy sets (Atanassov, 1994; Akram & Dudek, 2013; Beliakov, Bustince, Goswami, Mukherjee, & Pal, 2011; Xia, Xu, & Liao, 2013; Xu, 2013; Xu & Yager, 2011; Zhao, Xu, Ni, & Liu, 2010), vague sets (Gau & Buehrer, 1993), hesitant fuzzy sets (Chen, Xu, & Xia, 2013, 2015; Wu, Wang, Wang, Zhang, & Chen, 2014; Xu & Xia, 2011; Zhu & Xu, 2014), interval-valued hesitant fuzzy sets (Liu, Ju, & Yang, 2014; Wang, Wu, Wang, Zhang, & Chen, 2014;) and fuzzy multisets (Miyamoto, 2005).

As a significant human activity, multiple attribute decision making (*MADM*) problems (He, Chen, He, & Zhou, 2015; He, He, & Chen, 2015; Xia, Xu, & Zhu, 2013; Yager, 1988) are the process of finding the best alternative(s) from all of the feasible alternatives where all the alternatives can be evaluated according to a number of attributes. Information aggregation is one of the core techniques—many papers have investigated this issue (Dubois & Prade, 1980; Gau et al., 1993; He, Chen, Zhou, Liu, & Tao, 2014; He & He, 2016; He, Zhu, & Park, 2012; Narukawa, 2007; Torra, 2010; Wang & Dong, 2009; Wei, 2012; Xia & Xu, 2011; Xu & Xia, 2011; Yager, 2008; Zhou & Chen, 2011; Zhu & Xu, 2014; Zhu, Xu,

* Corresponding author.
E-mail addresses: 812256307@qq.com, yidhe@tju.edu.cn (Y. He), zhhe@tju.edu.cn (Z. He).

http://dx.doi.org/10.1016/j.cie.2016.07.004
0360-8352/

& Xia, 2012. Yager (2001) originally introduced the power average (*PA*) operator. Bonferroni (1950) considered the interrelationship of the individual arguments and introduced a mean-type aggregation operator called the Bonferroni mean (*BM*). Zhu et al. (2012) presented the hesitant fuzzy geometric Bonferroni means. Xia and Xu (2011) proposed some hesitant fuzzy aggregation operators. Wei (2012) developed some prioritized aggregation operators for aggregating hesitant fuzzy information. Zhang (2013) developed a series of hesitant fuzzy power aggregation operators. Liao, Xu, and Xia (2014) investigated the multiplicative consistency of a hesitant fuzzy preference relation. Rodriguez et al. (2012) presented the hesitant fuzzy linguistic term sets. Chen et al. (2013) introduced the concept of interval-valued hesitant fuzzy sets (*IVHFSs*), permitting the membership degrees of an element to a given set to have a few different interval values.

However, the existing ranking method (Zhang, Wang, Tian, & Li, 2014) for interval-valued hesitant fuzzy sets can't rank all *IVHFSs*. For example, assume $\tilde{h}_1$ and $\tilde{h}_2$ are two interval-valued hesitant fuzzy sets, $\tilde{h}_1 = \left\{\tilde{h}_1^1, \tilde{h}_1^2\right\}$, $\tilde{h}_2 = \left\{\tilde{h}_2^1, \tilde{h}_2^2\right\}$, $\tilde{h}_1^1 + \tilde{h}_1^2 = \tilde{h}_2^1 + \tilde{h}_2^2$, $\tilde{h}_1^1 \neq \tilde{h}_2^1$, $\tilde{h}_1^1 \neq \tilde{h}_2^2$, and $\tilde{h}_1^2 \neq \tilde{h}_2^1$, $\tilde{h}_1^2 \neq \tilde{h}_2^2$, if we use the comparison method by Zhang et al. (2014), i.e., just considering the score of different *IVHFSs*, then $h_1 = h_2$ for $s(h_1) = s(h_2)$. However, $h_1^1 \neq h_2^1, h_1^1 \neq h_2^2$, $h_1^2 \neq h_2^1, h_1^2 \neq h_2^2$, which means it is actually not reasonable to have the result that $h_1 = h_2$. If we take account the dispersive central polymerization degree of all values in the interval-valued hesitant fuzzy set, the above weakness can be solved. Therefore, we define the interval-valued hesitant fuzzy 2nd-order dispersive central polymerization degree function, which can be explained as the variance in statistics. Based on this, a new ranking method is presented. Moreover, in many real decision making problems, it may be difficult for decision makers to exactly quantify their opinions with a single crisp number due to the insufficiency in available information, but instead define an interval number in $[0,1]$. Motivated by Chen et al. (2013) and He, He, Wang, and Chen (2015), we present the *IVHFPBM* and the *IVHFWPBM*, capturing not only the interrelationship between input arguments, but also the relationships between the fusedvalues, providing a new train of thought for multiple attribute group decision making underinterval-valued hesitant fuzzy environments.

The rest of paper is organized as follows. Section 2 reviews some basic concepts and proposes the new ranking methods for different interval-valued hesitant fuzzy sets. Section 3 develops the *IVHFPBM* and the *IVHFWPBM*, investigates their desirable properties, and evaluates some special cases. Section 4 applies the new aggregation operators to interval-valued hesitant fuzzy multi-criteria group decision making. Section 5 investigates a numerical example to illustrate the feasibility and effectiveness of the new approaches. Section 6 ends the paper.

## 2. Preliminaries

In this section, we first define some important notations. Then we review the basic concepts, including the power average (*PA*) operator (Yager, 2001), interval-valued hesitant fuzzy sets (*IVHFSs*) (Chen et al., 2013) and some basic operational laws on interval-valued hesitant fuzzy elements (*IVHFEs*). Then we propose an improved comparison law for *IVHFEs*.

### 2.1. Important notations

Important notations required for the variables and formulas are provided below:

| | |
|---|---|
| $D[0,1]$ | The set of all closed subintervals of $[0,1]$ |
| $X$ | A fixed set |
| *PA* operator | Power average operator |
| *POWA* operator | Power ordered weighted average operator |
| *IVHFE* | Interval-valued hesitant fuzzy element |
| $\#\tilde{h}$ | The number of the elements in $\tilde{h}$ |
| $s(\tilde{h})$ | The score function of $\tilde{h}$ |
| $IVHFCP_2$ function | Interval-valued hesitant fuzzy 2nd-order central polymerization degree function |
| $IVHFDCP_2$ function | Interval-valued hesitant fuzzy 2nd-order dispersive central polymerization degree function |
| *IVHFPBM* | Interval-valued hesitant fuzzy power Bonferroni mean |
| $d(\tilde{h}_l, \tilde{h}_r)$ | The distance between $\tilde{h}_l$ and $\tilde{h}_r$ |
| $Supp(\tilde{h}_i, \tilde{h}_j)$ | The support for $\tilde{h}_i$ and $\tilde{h}_j$ |
| $\eta_{ij} \oplus \eta_{ji}$ | The bonding satisfaction factor used as a calculation unit, capturing the connection between $\tilde{h}_i$ and $\tilde{h}_j$ $i,j = 1,2,\ldots,n; i \neq j$ |
| *IVHFWPBM* | Interval-valued hesitant fuzzy weighted power Bonferroni mean |
| $x = \{x_1, x_2, \ldots, x_m\}$ | The set of alternatives |
| $g = \{g_1, g_2, \ldots, g_n\}$ | The set of attributes |
| $(w_1, \ldots w_n)$ | The associated weighting vector of attributes |
| $(\omega_1, \ldots \omega_k)$ | The associated weighting vector of experts |
| $y = \{y_1, \ldots, y_K\}$ | A group of experts. |

### 2.2. Basic concepts

Chen et al. (2013) pointed out that it may be difficult for decision makers (DMs) to exactly quantify their opinions with a crisp number due to insufficiency in available information in many real decision making problems, and introduced the concept of interval-valued hesitant fuzzy sets (IVHFSs), which permits the membership degrees of an element in a given set to have a few different interval values.

**Definition 1** Chen et al., 2013. Let $X$ be a fixed set, and $D[0,1]$ be the set of all closed subintervals of $[0,1]$. An interval-valued hesitant fuzzy set (*IVHFS*) on $X$ is $\tilde{E} = \{\langle x, h_{\tilde{E}}(x)\rangle | x \in X\}$, where $h_{\tilde{E}}(x)$ is a set of some intervals in $D[0,1]$, denoting the possible membership degree intervals of the elements $x \in X$ to the set $\tilde{E}$.

For convenience, Chen et al. (2013) and Zhang et al. (2014) called $\tilde{h} = h_{\tilde{E}}(x)$ an interval-valued hesitant fuzzy element (*IVHFE*) and $\tilde{H}$ the set of all *IVHFEs*. For the element $\gamma \in \tilde{h}$, $\gamma = [\gamma^L, \gamma^U]$, where $0 \leqslant \gamma^L \leqslant \gamma^U \leqslant 1$.

Chen et al. (2013) defined systematic aggregation operators to aggregate interval-valued hesitant fuzzy information for decision making problems under interval-valued hesitant fuzzy environments, considering the differences of opinions between different individual decision makers.

**Definition 2.** As stated by Chen et al. (2013), given three *IVHFEs* $\tilde{h}$, $\tilde{h}_1$, $\tilde{h}_2$, and $\lambda > 0$. Some basic operations on them were defined as follows.