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Shortest path problem of uncertain random network

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ABSTRACT

The shortest path problem is one of the most fundamental problems in network optimization. This paper is concerned with shortest path problems in non-deterministic environment, in which some arcs have stochastic lengths and meanwhile some have uncertain lengths. In order to deal with path problems in such network, this paper introduces the chance theory and uses uncertain random variable to describe non-deterministic lengths, based on which two types of shortest path in uncertain random networks are defined. To obtain the shortest paths, an algorithm derived from the Dijkstra Algorithm is proposed. Finally, numerical examples are given to illustrate the effectiveness of the algorithms.

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1. Introduction

As one of the fundamental problems in network optimization, the shortest path problem concentrates on finding a path with minimum distance, time or cost from the source to the destination. Some other optimization problems, such as transportation, communications and supply chain management, can be considered as special cases of the shortest path problem. Since the late 1950s, the shortest path problem and problems stemming from it have been widely studied, and some successful algorithms have been proposed, such as Bellman (1958), Dijkstra (1959), Dreyfus (1969), and Floyd (1962).

In traditional networks, the length(weight) of each arc is deterministic. However, because of failure, maintenance, or other reasons, arc lengths are non-deterministic in many situations. Some researchers believed that these non-deterministic phenomena conform to randomness, so they introduced probability theory into network optimization problems and used random variables to describe the non-deterministic lengths. Random network was first investigated by Frank and Hakimi (1965) in 1965 for modeling communication networks with random capacities. From then on, random networks have been well developed and widely applied. Many researchers have done lots of work on random shortest path problems, such as Frank (1969), Nie and Wu (2009), Chen, Lam, Sumalee, and Li (2012), and Zockaie, Nie, and Mahmassani (2014). In these literature, the wights of arcs were regarded as random variables, and corresponding models were studied. However, such probability-based methods can effectively work only when probability distribution functions of non-deterministic phenomena are exactly obtained. That is to say, if there are not enough observational data to support the non-deterministic phenomena, the probability distribution functions estimated via statistical methods will not be close to real situations, which indicates that it is not suitable to employ random variables to model non-deterministic phenomena in these cases.

In the cases that there is little or no observational data for non-deterministic phenomena, a feasible and economic way is to estimate the data by experts based on their subjective information and experiences. As a matter of course, additivity and some other specific properties of probability calculus fall away when there are only expert empirical data, since human factors are so imprecise and there is almost no rigorous additive property can be discovered (Kóczy, 1992). To describe expert data in arc lengths, uncertainty theory (Liu, 2007, 2010) was introduced into network optimization problems and uncertain variables were employed to model arc lengths. Uncertain network was first explored by Liu (2009) for modeling project scheduling problem with uncertain duration times. In 2011, Gao (2011) deduced the uncertainty distribution function of the shortest path length, and studied the α-shortest path and the most measure shortest path in uncertain networks. In 2014, Han, Peng, and Wang (2014) studied maximum flow problems in networks with uncertain capacities, and designed a 99-algorithm to calculate the uncertainty distribution function of maximum flow. In 2015, Gao, Yang, Li, and Kar (2015) and Gao and Qin (in press) investigated uncertain graphs, and proposed efficient algorithms to calculate the distribution function of the diameter and the edge-connectivity of an uncertain graph. Besides, the

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uncertain minimum cost flow problem was investigated by Ding (2014), and the Chinese postman problem was studied by Zhang and Peng (2012).

In some real cases, uncertainty and randomness simultaneously appear in a system. Specifically, for some non-deterministic phenomena, we have enough observational data to obtain their probability distribution functions; while for others, we can only estimate them by expert data. In order to deal with this kind of systems, Liu (2013) proposed chance theory in 2013. Further, Liu (2014) introduced the chance theory into networks, which involves both random arc lengths and uncertain arc lengths. In the paper, Liu proposed a concept of uncertain random network, in which the lengths of some arcs are random variables and other lengths are uncertain variables. Furthermore, Liu studied the chance distribution function of minimum length from source node to destination node in uncertain random networks. Besides, models for maximum flow problems in uncertain random networks were constructed by Sheng and Gao (2014).

This paper will further study the shortest path problem within the framework of chance theory. The contributions of this paper are threefold. First, we propose an algorithm, which is derived from Dijkstra Algorithm, to calculate the chance distribution function of the minimum length from source node to destination node in an uncertain random network. Note that in Liu (2014), only a theoretical formula was proposed to calculate the chance distribution function of the minimum length, which is difficult to use in practice. The proposed algorithm in this paper numerically calculates the chance distribution function.

Second, two types of shortest paths in uncertain random networks are first proposed, which are simply named as *type-I shortest path* and *type-II shortest path*, respectively. In an uncertain random network, the length of a path is an uncertain random variable, instead of a constant. As a result, the traditional concept of shortest path is not suitable in uncertain random networks any more. The definitions of type-I and type-II shortest paths are both based on minimizing deviations between the chance distribution function of the length of a path and the chance distribution function of the minimum length from source node to destination node. The deviations between chance distribution functions are presented in Section 4 in detail.

Third, optimization models are respectively constructed to model the proposed types of shortest paths. The properties of the models are investigated, based on which an algorithm is developed to solve the model. The effectiveness of the algorithm is illustrated by several numerical examples.

It should be pointed out that in this paper, we don't distinguish risk-averse decision-makers and risk-prone users decision-makers, that is, we assume that all the decision-makes have the same perspective on uncertainty. Besides, all the uncertain lengths and random lengths are assumed to be independent throughout this paper. The remainder of this paper is organized as follows. In Section 2, some basic concepts of uncertainty theory and chance theory used throughout this paper are introduced. Section 3 proposes an algorithm to calculate the chance distribution function of the minimum length from the source node to the destination node in uncertain random networks. In Section 4, two types of shortest paths in uncertain random networks are proposed and investigated in details, and then an algorithm is designed to search the corresponding paths. In Section 5, a numerical example is given to illustrate the proposed model and algorithm. Section 6 gives a brief summary to this paper.

2. Preliminaries

Recall that uncertainty theory and probability theory are two different mathematical tools to describe and model non-deterministic phenomena. Specifically, if we have enough historical or experimental data for the non-deterministic phenomena, then these phenomena can be described by random variables; while if there are only expert empirical data to estimate the non-deterministic phenomena, it is better to use uncertain variables to describe these phenomena.

In this section, we first review some concepts of uncertainty theory, including uncertain measure, uncertain variable and operational law. Then we introduce some useful definitions and properties about chance theory, such as uncertain random variable and chance distribution function.

2.1. Uncertainty theory

Let Γ be a nonempty set, and $\mathcal{L}$ be a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is designated a number $\mathcal{M}\{\Lambda\}$. Liu (2007) proposed four axioms to ensure that the set function $\mathcal{M}\{\Lambda\}$ satisfy certain mathematical properties.

Axiom 1 (*Normality*). $\mathcal{M}\{\Gamma\} = 1$.

Axiom 2 (*Duality*). $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ.

Axiom 3 (*Subadditivity*). For every countable sequence of events $\{\Lambda_i\}$, we have

$$\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \leqslant \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}.$$

Axiom 4 (*Product Axiom*). Let $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i)$ be uncertainty spaces for $i = 1, 2, \ldots$. Then the product uncertain measure $\mathcal{M}$ is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{i=1}^{\infty}\Lambda_i\right\} = \bigwedge_{i=1}^{\infty}\mathcal{M}_i\{\Lambda_i\},$$

where $\bigwedge$ is the maximum value operator, and Λ_i are arbitrarily chosen events from $\mathcal{L}_i$ for $i = 1, 2, \ldots$, respectively.

Note that the first three axioms of uncertainty theory also hold in probability theory, while the product axiom of uncertainty theory is totally different from that of probability theory, which leads to the difference of these two theories.

To describe a quantity with uncertainty, uncertain variable is defined analogous to the definition of random variable.

Definition 1 Liu (2007). An uncertain variable ξ is a measurable function from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

Let $x \in \Re$ and $B = (-\infty, x]$. Then $\Phi(x) = \mathcal{M}\{\xi \in B\} = \mathcal{M}\{\xi \leqslant x\}$ is called the uncertainty distribution function of ξ. The inverse function Φ^{-1} is called the inverse uncertainty distribution function of ξ if it exists and is unique for each $\alpha \in (0, 1)$. Inverse uncertainty distribution function plays a crucial role in operations of independent uncertain variables.

The following theorem presents the operational law of independent uncertain variables, which gives the uncertainty distribution function of $\xi = f(\xi_1, \xi_2, \ldots, \xi_n)$.

Theorem 1 Liu (2010). *Let $\xi_1, \xi_2, \ldots, \xi_n$ be independent uncertain variables with uncertainty distribution functions $\Phi_1, \Phi_2, \ldots, \Phi_n$, respectively. If f is strictly increasing function, then*

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