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High quality process monitoring using a class of inter-arrival time distributions of the renewal process

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ABSTRACT

For high-quality processes where the defect rate is very low, e.g., parts per million (ppm), time-between-events (TBE) control charts have several advantages over the ordinary control charts. Most existing TBE control charts are based on the homogeneous Poisson process assumption, so that the distribution of TBE can only be exponential. However, the exponential distribution is not suitable in many applications, especially when the failure rate is not constant. In this article, we introduce a new TBE control chart, based on the renewal process, where the distribution of the TBE belongs to a parametric class of absolutely continuous distributions, which includes some well-known and commonly used lifetime distributions, i.e., exponential, Rayleigh, Weibull, Burr type XII, Pareto and Gompertz. The control structure of the proposed chart is derived analytically in general and numerical examples are presented for the Weibull distribution, due to its relevance in reliability. The performance of the proposed control charts is evaluated in terms of some standard useful measures, including average run length (ARL), the standard deviation of run length, the coefficient of variation of run length, expected quality loss (EQL) and relative ARL (RARL). The effect of parameter estimation, using both maximum likelihood and Bayesian methods, is also discussed. This study also presents an illustrative example and four case studies to highlight the practical aspects of the new TBE chart.

1. Introduction

A manufacturing process ability to meet specifications depends on two factors: variation and accuracy. Variation is relative to the specification width, and the process capability indices are used to measure this relationship, while accuracy implies that the process mean is at the nominal level. The purpose of the control chart is to detect undesirable changes in the process as early as possible. The choice of an appropriate control chart is a debatable point and conclusions are affected by many key factors. There are two main types of available control charts: attribute and variable control charts. To monitor the fraction of nonconformities of a process, attribute control charts like p and np, c and u, or r, are the well-known charts when the numbers of defects in a sample follows the binomial, the Poisson, or the negative binomial distribution, respectively. However, for high quality processes with a very low defect rate (i.e. parts per million or per billion, especially in the fields of manufacturing of integrated circuits, weapon systems, automobile engine and many other automated processes) these charts have certain drawbacks, i.e., high false alarm rates, negative values of lower control limits for positive monitored quantities (cf. Table 1), undesirable dependencies between sample size and control limits (when admissibility of the latter is enforced) and poor approximation to the normal distribution (cf. Chan, Dennis, Xie, & Goh, 2002). For example, if we construct (approximate) control chart limits based on the Poisson (P), binomial (B), negative binomial (NB) and zero-inflated Poisson (ZIP) distributions, then upper control limits (UCL) and lower control limits (LCL) are given as below:

$$\lambda \pm k\sqrt{\frac{\lambda}{n}} \tag{1}$$

$$np \pm k\sqrt{np(1-p)} \tag{2}$$

$$\frac{r(1-p)}{p} \pm k\sqrt{\frac{r(1-p)}{np^2}} \tag{3}$$

$$(1-\pi)\lambda \pm k\sqrt{\frac{\lambda(1-\pi)(1+\pi\lambda)}{n}} \tag{4}$$

where $p, r, \pi, \lambda > 0$ and k determine the confidence level, which is usually set to 99.73% (corresponding to $k = 3$, the so-called 3σ

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Table 1
Values of n when $k = 3$ for various distributions of the monitoring statistics.

Distribution	p (ppm)					
	100	400	800	5000	10,000	200,000
P	90,000	22,500	11,250	1800	900	45
B	89,991	22,491	11,241	1791	891	36
NB (GM-$r = 1$)	90,000	22,500	11,250	1800	900	45
NB ($r = 10$)	9000	2250	1125	180	90	5
ZIP ($\pi = 0.2$)	112,502	28,127	14,065	2252	1127	59
ZIP ($\pi = 0.4$)	150,006	37,506	18,756	3006	1506	81

limits of the normal approximation). To avoid the LCL to be negative, the following conditions for k, given n, must be satisfied: $k \leqslant \sqrt{n\lambda}$, $k \leqslant \sqrt{np/(1-p)}$, $k \leqslant \sqrt{nr(1-p)}$ and $k \leqslant \sqrt{\frac{n\lambda(1-\pi)}{1+\pi\lambda}}$, respectively, making the desired false alarm rate unattainable if the normal approximation is poor. Alternatively, one could increase the sample size, obtaining conditions $n \geqslant k^2/\lambda$, $n \geqslant k^2(1-p)/p$, $n \geqslant \frac{k^2}{r(1-p)}$ and $n \geqslant \frac{k^2(1+\pi\lambda)}{\lambda(1-\pi)}$. Table 1 shows the sample sizes for the commonly used 3σ control limits as p gets smaller. Clearly, one needs impractical sample sizes to monitor nonconformities effectively.

Alternatively, one could try to use only one-sided charts, controlling whether the monitoring statistics is larger than the UCL. For the np-chart, for example, the upper 99.73% control limits with $p = 0.01$, as $n = 5$, 10, 20, 50 and 100 are 1, 2, 2, 3 and 5. But, because of the discreteness of the binomial distribution, the upper control limit would be meaningless for very small p, e.g., for $p = 10^{-4}$ and $n = 5$, the upper control limit is zero.

Thus, traditional process monitoring techniques for count data are not sufficient for high quality processes. In such situations, the time between events (TBE) chart is an efficient approach for monitoring, controlling and improving the process when the event occurrence rate is very low. Here, the word "event" usually refers to the occurrence of nonconforming items or of defects in the manufacturing process, whereas "time" is the conforming run length for discrete processes or the product quantity between two consecutively observed defects for continuous processes.

The available TBE control charts can be categorized into two groups: attribute TBE and variable TBE. Most of the attribute TBE charts are based on the geometric distribution (cf. Ali, Pievatolo, & Göb, 2016), such as the cumulative count control (CCC) chart, or on the negative Binomial distribution (e.g. the CCC-r chart). One special variable TBE chart is the cumulative quantity control (CQC) chart. As the occurrence of the events follows a Poisson process, the time between two events follow an exponential distribution, so CQC can also be called exponential chart, sometimes denoted as t-chart. Calvin (1983) proposed the first CCC chart based on the geometric distribution to monitor high-quality processes while Nelson (1994) and Goh (1987) gave a detailed discussion about its implementation. Xie, Goh, and Ranjan (2002) studied some properties of the CCC, CQC and CQC-r charts. Cheng and Chan (2010) proposed CCC-r chart based on the negative Binomial distribution which is considered as an improvement to the existing CCC chart. Later, Chan et al. (2002) proposed the cumulative probability control (CPC) charts based on the geometric and exponential distributions. Due to the popularity and simplicity of the Poisson process, CQC charts are used in various applied fields like the monitoring of the accident rate in a transportation system, the rate of occurrence of congenital malformations or the volume of paperwork between errors, etc. Zhang, Xie, and Goh (2006) introduced an exponential control chart based on the sequential sampling scheme with the self starting feature where the defect rate follows a Poisson process. Shamsuzzaman, Xie, Goh, and Zhang (2009) developed an economic model for the exponential chart to monitor time-between-events data. Some recent contributions to the TBE monitoring are: exponential TBE control charts using the repetitive sampling concept proposed by Aslam, Khan, Azam, and Jun (2014), the Gumbel bivariate exponential distribution control chart proposed by Xie, Xie, and Goh (2011), TBE control charts using different sampling schemes proposed by Qu, Wu, Khoo, and Rahim (2014), variable sampling interval and variable limits control charts by Chen, Chen, and Chiou (2011). We refer to Ali et al. (2016) for a comprehensive overview of TBE charts in high-quality processes.

Most variable TBE control charts are based on the homogenous Poisson process assumption (cf. Ali et al., 2016), so that the distribution of TBE can only be assumed exponential. However, the exponential distribution is not suitable in many applications, especially when the failure rate is not constant. The major aim of the article is to generalize the available TBE charts to situations where one could have an increasing, decreasing, bathtub or monotone failure rate. Therefore, we propose the use of a renewal process to generalize the existing homogenous Poisson TBE control charts. In this article, we consider the development of a control chart based on the renewal process where the distribution of time is assumed to belong to a class of absolutely continuous distributions. This class includes exponential, Rayleigh, Weibull, Burr type XII, Pareto and Gompertz distributions. A renewal process can be regarded as a generalization of the ordinary Poisson process, where the exponential distribution of time between occurrences is replaced by any other lifetime distribution.

Definition 1.0.1. A counting process $\{N(t), t \geq 0, t \in T\}$ with independent and identically distributed (iid) inter-arrival times $X_1, X_2, \ldots$ having a common distribution F is called a renewal process.

The use of a renewal process is motivated by TBE with a non-constant hazard rate. Consider, for example, an industrial process in which a sensor signals that a filter must be substituted. The hazard of getting a substitution signal increase with time because impurities trapped within the filter accumulate. After changing the filter a renewal takes place, represented by a new filter with the same hazard function. An increased renewal frequency with respect to the nominal would indicate an upstream problem in the process because filters are changed too often. Renewal theory has many applications, especially in the field of repairable systems, component testing, the time intervals of successive earthquakes in a particular region and so on.

Thus, the present study is a generalization of the existing TBE charts. This article also includes a study of the performance of the proposed TBE control chart from different perspectives. We derive explicit expressions of the control limits and of the ARL for the members of the above-mentioned class of distributions. Then we run numerical examples, where we compute and compare ARLs for a selection of parameter shifts. To evaluate the overall performance of control charts, the EQL and RARL are also computed. Due to its effectiveness and wide usage, we carry out further analyses on the Weibull chart: we illustrate simulated and real data applications and we also include a study of the control chart performance in presence of parameter estimation error.

The rest of the article is organized as follows: in Section 2, the class of distributions is defined. The design of the TBE chart for high yield processes is discussed in Section 3. In Section 4, it is explained how the proposed methodology can be used in real situations with the help of four illustrative examples. In Section 5, some performance criteria for the evaluation of control chart performances are presented. They are: average run length, expected quadratic loss and relative average run length. The Bayesian and