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Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts

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ABSTRACT

In typical EOQ-based inventory models, the demand rate and the holding cost are assumed to have constant values and the unit purchase cost is assumed constant regardless of the order size. In actual applications, however, the demand rate for a specific item can be affected by many variables such as seasonality, selling price, and availability. Moreover, the unit holding cost tends to be higher for extended storage periods. Additionally, the unit purchase cost is generally lower for larger order sizes due to quantity discounts. The objective of this paper is to simultaneously consider the variability of the demand rate, the unit holding cost, and the unit purchase cost. An inventory model is presented with a selling price-dependent demand rate, a storage time-dependent holding cost, and an order size-dependent purchase cost based on all-units quantity discount. A mathematical model is constructed, and a solution methodology is developed for determining the optimal solution. A numerical example is solved, and sensitivity analysis is conducted to study the effect of various parameters on the optimal solution.

1. Introduction

The majority of inventory control models in the literature are based on the assumption that the demand rate is a fixed constant over the entire inventory cycle. Most of these models consider the demand rate as an independent exogenous variable, which is outside the control of the given organization. However, in real life, the demand rate for a specific item can be affected by many variables such as the sale price and the availability of the item. Inventory models with price-dependent demand rates are based on the fact that lower selling prices tend to increase sales for many products. Moreover, in practice, the unit holding cost may vary over the storage duration because of the time value of money and the length of the warehouse lease contract. In variable holding cost models, the unit holding cost is assumed to increase either continuously or discretely as a function of the time spent in stock.

The objective of this paper is to develop an inventory model where the demand rate depends on the selling price, the unit holding cost depends on the storage time, and the purchase cost depends on the order size. In the proposed model, the demand rate is assumed to be a linearly decreasing continuous function of the unit selling price. The unit holding cost is expressed as a linearly increasing continuous function of the storage time. The unit purchase cost is a decreasing discontinuous (step) function of the order quantity according to an all-units quantity discount scheme. Based on these assumptions, a mathematical model is formulated, and an optimal solution methodology is developed to maximize the total profit.

The above-mentioned set of assumptions, which has not been considered previously in the literature, represents several practical real-life situations. A typical example of industries that actually operate under the same set of assumptions is the food industry. With longer storage durations, many processed food items require more sophisticated warehousing facilities, and therefore time-increasing holding costs. Moreover, demand for food items tends to be price-dependent, partly because these items are generally not unique. Therefore, if the price of these items increases, customers usually switch to other brands or even to alternative food items. Finally, quantity discounts are often offered on many food items to stimulate demand and avoid extended storage times.

The remainder of this paper is organized as follows. Relevant literature is reviewed in Section 2. The problem is defined and the model is constructed in Section 3. The solution algorithm is presented and a numerical example is solved in Section 4. Finally, suggestions and concluding remarks are provided in Section 5.

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2. Literature review

In this section, the most relevant recent research is classified into four types of economic order quantity (EOQ) inventory models. The first type is models with variable demand. The second type is models with variable holding costs. The third type is models with both variable demand and variable holding costs. The fourth type is models with quantity discounts.

2.1. Inventory models with variable demand

In inventory models with variable demand rates, the demand for the given item is assumed to vary as a function of either the price, the stock level, or both.

Inventory models in which the demand rate depends on item availability or stock level are very common. Sana and Chaudhuri (2004) constructed an EOQ model in which the demand rate depends on both item availability and advertising expenditures. The model maximizes the net profit while satisfying budget and storage-capacity constraints. Min and Zhou (2009) proposed an inventory model for deteriorating items with a stock level-dependent demand, partial backlogging, and a limit on the maximum inventory level. Yang, Teng, and Chern (2010) developed another inventory model with stock-dependent demand for deteriorating items, allowing partial backlogging and considering the effect of inflation. Lee and Dye (2012) formulated an EOQ model with partial backlogging, stock-dependent demand, and a controllable deterioration rate. The model determines the optimal ordering and preservation (deterioration control) policies to maximize the total profit.

Many inventory models assume higher demand with lower prices. Mondal, Bhunia, and Maiti (2003) developed an inventory model for deteriorating products assuming the demand rate is a linear function of the selling price, and the deterioration rate is a decreasing function of the storage duration. Similarly, Mukhopadhyay, Mukherjee, and Chaudhuri (2005) investigated an inventory model for a deteriorating item where the demand rate varies with the selling price and the deterioration rate varies with the storage time. Assuming a price-dependent demand rate, Teng, Chang, and Goyal (2005) developed an EOQ model to determine the price and the lot size for a retailer when the supplier allows delayed payments. Transchel and Minner (2008) developed optimal solution algorithms for an EOQ model with quantity discounts and price-dependent demand, considering the linear price function as a special case. Different strategies were considered, with and without centralization (determining the selling price and the lot size simultaneously) and dynamic pricing (changing selling price). Zhengping (2010) studied an inventory model with a price-dependent demand, given partial demand information, in a supply chain with one retailer and one supplier.

Several models assume the demand rate depends on both the stock level and the selling price. You (2004) developed an inventory model to determine the optimal ordering quantity and selling price, assuming a time- and price-dependent demand rate. Hou and Lin (2006) proposed an inventory model for deteriorating items where the demand rate is dependent on both the selling price and stock-level, taking the time value of money into consideration to maximize the net present value of profit. You and Hsieh (2007) studied an inventory system in which the demand rate is a function of both the selling price and item availability. Panda, Maiti, and Maiti (2010) analyzed an EOQ model for one vendor and multiple retailers for an item whose demand rate is a linear function of both the stock level and the selling price.

2.2. Inventory models with variable holding cost

Many inventory models assume the unit holding cost to be variable. Ferguson, Jayaraman, and Souza (2007) developed an inventory model in which the holding cost has non-linear dependence on the storage time. Assuming the value of the item decreases non-linearly with storage time, the model considered delivery surcharges for infrequent orders and price discounts for perishable products. Ghasemi and Afshar Nadjafi (2013) developed two EOQ models with variable holding costs, one with and the other without backorders. The holding cost is assumed to be constant until a given time period, and then it becomes an increasing function of the ordering cycle length. San-José, Sicilia, and García-Laguna (2015) studied an EOQ model with partial backordering where the holding cost function has two components: a fixed cost, and a variable cost that increases with storage time.

2.3. Inventory models with variable demand and variable holding cost

In several models in which both the holding cost and the demand rate are variable, the demand rate is assumed stock-dependent. Alfares (2007) proposed an EOQ-based model with stock-level dependent demand rate, where the holding cost increases either retroactively or incrementally with longer storage times. Alfares (2014) extended this model to an EPQ model, with finite production rate, non-zero starting inventory, and profit-maximization objective. Zhao and Zhong (2008) modified the model of Alfares (2007) by assuming the holding cost to be a decreasing function of the storage time. Pando, García-Laguna, San-José, and Sicilia (2012) studied an EOQ model in which the demand rate and the holding cost depend on the stock level. Pando, San-José, García-Laguna, and Sicilia (2013) maximized the total profit for an inventory model with inventory level-dependent demand. The holding cost was assumed a non-linear function of both the storage time length and the inventory level.

Several inventory models considered demand dependence on other factors such as the product selling price and quality. Datta (2013) analyzed an inventory model assuming that the demand depends on both the quality and the selling price of the products. Kumar, Chauhan, and Kumar (2013) studied an EOQ model under the assumption of price-dependent demand, where the holding cost is a time-function of the trade credit for deteriorating items. Datta and Paul (2001) studied a similar EOQ model assuming the demand rate depends on both the inventory stock and the price of the item, while the holding cost varies with the storage time.

2.4. Inventory models with quantity discounts

Inventory models in which the purchase cost depends on the order size are based on either all-units or incremental quantity discounts. In all-units discounts, one reduced price applies to all the units in the given order. In incremental discounts, successively decreasing prices apply to subsets of units above certain specified limits. Weng (1995) developed an inventory model to determine optimal quantity discount policies, assuming the demand depends on the selling price. The model is used to maximize profits, considering both all-units and incremental quantity discounts, where shortages are not allowed and the holding cost is constant. Burwell, Dave, Fitzpatrick, and Roy (1997) developed an inventory model to determine the optimal lot size and selling price, assuming the demand is price-dependent, the holding cost is a function of the total unit cost, and both freight and all-unit quantity discounts are offered. Chang (2013) modified the model and the algorithm of Burwell et al. (1997) in order to maximize the profit and determine the exact optimal values of the lot size and the selling price.