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Phase II monitoring of generalized linear profiles using weighted likelihood ratio charts

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A B S T R A C T

In recent years, effective profile monitoring for discrete response variables, such as binary, multinomial, ordinal or Poisson variables, has increasingly attracted interest of researchers in the area of statistical process control. Such quality characteristics are often modeled as special cases of generalized linear models. The objective of this paper is to try to provide a unified framework for Phase II monitoring of generalized linear profiles of which the explanatory variables can be fixed design or random arbitrary design. To this end, a new control chart is developed based on the weighted likelihood ratio test, and it can be readily extended to other generalized profiles or profiles with random predictors if the likelihood function can be obtained. Numerical results and illustrative example show that the proposed control chart has satisfactory in-control run length distribution and stands out at early detection.

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1. Introduction

Statistical profile monitoring has increasingly attracted researchers' attention in the area of statistical process control. Early reviews of work in profile monitoring include Woodall, Spitzner, Montgomery, and Gupta (2004) and Woodall (2007), and a recent comprehensive review Woodall and Montgomery (2014) recommend Noorossana, Saghaei, and Amiri (2011) for a more up-to-date overview as the chapters in this book were written by some of the leading researchers in profile monitoring. For profile monitoring, one group of monitoring methods are interested in the case that the response variables are continuous (e.g., Huwang, Wang, Xue, & Zou, 2014; Li & Wang, 2010; Zou, Ning, & Tsung, 2012). At the meantime, it is also quite common to deal with profile monitoring with discrete response variables. As far as we know, the pioneering work is Yeh, Huwang, and Li (2009). Some recent work, such as Amiri, Koosha, and Azhdari (2011), Noorossana, Aminnayeri, and Izadbakhsh (2013), Noorossana, Saghaei, Izadbakhsh, and Aghababaei (2013) and Soleymanian, Khedmati, and Mahlooji (2013), focused on profile monitoring whose response variables are Poisson, ordinal, multinomial and binary variables, respectively.

In the case of discrete response variables, the quality characteristics are often modeled as special cases of generalized linear models (GLM). Amiri, Koosha, Azhdari, and Wang (2015) and Shadman, Mahlooji, Yeh, and Zou (2015) provided a unified framework for Phase I control of generalized linear profiles. Besides the GLM, other types of models have also been used to represent profiles, such as simple linear regression (e.g., Aly, Mahmoud, & Woodall, 2015; Noorossana, Eyvazian, & Vaghefi, 2010; Zhang, Li, & Wang, 2009), nonlinear regression (e.g., Chang & Yadama, 2010; Paynabar, Jin, & Pacella, 2013), multiple regression (e.g., Eyvazian, Noorossana, Saghaei, & Amiri, 2011; Mahmoud, Saad, & El Shaer, 2015), nonparametric regression (e.g., Chuang, Hung, Tsai, & Yang, 2013; Qiu, Zou, & Wang, 2010), mixed models (e.g., Jensen & Birch, 2009; Koosha & Amiri, 2013), and wavelet models (e.g., Chicken, Pignatiello, & Simpson, 2009; Lee, Hur, Kim, & Wilson, 2012). All of the afore-mentioned research, however, only consider the case in which the explanatory variables are fixed from profile to profile. Shang, Tsung, and Zou (2011) provided an aluminium electrolytic capacitor example to illustrate the case in which different profiles often have random explanatory variables and these variables require careful monitoring as well. The major objective of this paper is to try to provide a unified framework for Phase II monitoring of generalized linear profiles of which the explanatory variables can be fixed design or random arbitrary design from profile to profile (the monitoring of the explanatory variables is not concerned). In Phase II, we are interested in

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detecting shifts in the model parameters as quickly as possible, while in Phase I, the purpose is to check the quality of historical data and to obtain accurate estimates of the model parameters.

In this paper, we developed a new control chart for generalized linear profile monitoring, which is based on the weighted likelihood ratio test (WLRT). Our proposed approach can be readily extended to other general profiles or profiles with random predictors if the likelihood function can be obtained. Other likelihood ratio test (LRT) based approaches can be found in Shang et al. (2011), Noorossana, Saghaei, et al. (2013) and Soleymanian et al. (2013). The exponentially weighted moving average (EWMA)-GLM control chart proposed by Shang et al. (2011) made use of all available profile samples up to the current time for estimating parameters, and different profiles are weighted as in an EWMA chart. Nevertheless, we found that the EWMA-GLM control chart has very large short-run false alarms, which renders this chart less useful and unacceptable in practice. Shewhart-type control charts (LRT) were proposed by Noorossana, Saghaei, et al. (2013) and Soleymanian et al. (2013). Another EWMA-type control chart (LRT-EWMA) was proposed by Soleymanian et al. (2013). It is shown that the Shewhart-type LRT control charts perform better at detecting large shifts, while the LRT-EWMA control charts perform better at detecting small to medium shifts. However, compared with our WLRT chart, the LRT-EWMA control chart was not as efficient due to the reason that it only used the current profile samples for estimating parameters, and thus the estimators would have considerably large bias and variance. Numerical results show that our proposed WLRT control chart has satisfactory in-control (IC) run length (RL) distribution and stands out at early detection, where RL is the number of points that must be plotted before a point indicates an out-of-control (OC) condition (Montgomery, 2013).

Now we summarize some abbreviated expressions used in this paper for easy reference.

IC	in-control
OC	out-of-control
RL	run length
ARL	average run length
SDRL	standard deviation of the run length
RMI	relative mean index
CED	conditional expected delay
EWMA	exponentially weighted moving average
MEWMA	multivariate exponentially weighted moving average
GLM	generalized linear models
LRT	likelihood ratio test
WLRT	weighted likelihood ratio test

The remainder of this paper is organized as follows. Our proposed methodology is described in detail in Section 2, including the statistical model and WLRT control chart. Section 3 is devoted to comparing the performance of five methods: WLRT, EWMA-GLM (Shang et al., 2011), LRT (Noorossana, Saghaei, et al., 2013; Soleymanian et al., 2013), LRT-EWMA (Soleymanian et al., 2013) and multivariate EWMA (MEWMA) (Soleymanian et al., 2013) charts. An illustrative example is given in Section 4. Section 5 concludes this paper and gives further discussion. The algorithm for obtaining the maximum weighted likelihood estimator is summarized in Appendix A.

2. The proposed WLRT scheme

In this section, we closely follow the notation and formulation used in Dobson (2002) to briefly discuss the generalized linear profiles. We assume that the observations are independent within and between profiles.

2.1. The statistical model

At any time point t, for the ith profile, our statistical model has three components:

1. Response variables $\widetilde{Y}_i = (Y_{i1}, \ldots, Y_{iN})^T$ share the same distribution from the exponential family with a canonical form,
$$f(y_{ij}; \theta_{ij}) = \exp[y_{ij} b(\theta_{ij}) + c(\theta_{ij}) + d(y_{ij})],\ i = 1, \ldots, t,\ j = 1, \ldots, N,$$
where $b(\cdot)$, $c(\cdot)$ and $d(\cdot)$ are known functions and θ_{ij}'s are the parameters of the exponential family of distributions.
2. Explanatory variables
$$\widetilde{X}_i = \begin{pmatrix} X_{i1}^T \\ \vdots \\ X_{iN}^T \end{pmatrix} = \begin{pmatrix} x_{i11} & \ldots & x_{i1p} \\ \vdots & & \vdots \\ x_{iN1} & \ldots & x_{iNp} \end{pmatrix},$$
where $X_{ij}^T = (x_{ij1}, \ldots, x_{ijp})$, $i = 1, \ldots, t$, $j = 1, \ldots, N$, can be combined linearly with a coefficient vector $\beta = (\beta_1, \ldots, \beta_p)^T$ (where $p < N$) to form the linear predictor $\eta_{ij} = X_{ij}^T \beta$.
3. A monotone link function $g(\cdot)$ such that
$$g(\mu_{ij}) = \eta_{ij} = X_{ij}^T \beta,\ i = 1, \ldots, t,\ j = 1, \ldots, N,$$
where $\mu_{ij} = E(Y_{ij})$.

Here, the explanatory variables $\widetilde{X}_i$ can be fixed design or random design from profile to profile. We suppose β changes from β_{IC} to another unknown value β_{OC} immediately after an unknown time point τ, which suffices to test the following hypotheses

$$\begin{cases} H_0 : \beta = \beta_{IC}, \\ H_1 : \beta \neq \beta_{IC}, \end{cases}$$

at each time point. Note that β_{IC} can be assumed known for Phase II monitoring.

2.2. Some existing work

From Dobson (2002), we know that, for the ith profile, the log-likelihood function is

$$l_i(\beta) = \sum_{j=1}^{N} [y_{ij} b(\theta_{ij}) + c(\theta_{ij}) + d(y_{ij})].$$

To obtain the maximum likelihood estimator of β, we can use the following estimating equation

$$\mathbf{b}_i^{(m)} = \mathbf{b}_i^{(m-1)} + [\mathfrak{I}_i^{(m-1)}]^{-1} U_i^{(m-1)},$$

where $\mathbf{b}_i^{(m)}$ is the vector of estimates of β at the mth iteration, $[\mathfrak{I}_i^{(m-1)}]^{-1}$ is the inverse of the information matrix, $U_i^{(m-1)}$ is the vector of score. When the difference between successive approximations $\mathbf{b}_i^{(m-1)}$ and $\mathbf{b}_i^{(m)}$ is sufficiently small, $\mathbf{b}_i^{(m)}$ is taken as $\hat{\beta}$ (maximum likelihood estimator of β).

Now we briefly review the LRT, LRT-EWMA and MEWMA control charts, which were proposed by Soleymanian et al. (2013) to monitor binary response profiles in Phase II. In fact, the LRT monitoring statistic can be expressed as

$$LRT_i = 2[l_i(\hat{\beta}_i) - l_i(\beta_{IC})],\ i = 1, 2, \ldots.$$

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