Permutation flowshop group scheduling with position-based learning effect

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**Abstract**

For the past few years, scheduling with learning effect has been receiving wide attention. However, learning effect and group technology have not been simultaneously explored in a flowshop setting although group technology plays an important role in a modern manufacturing system. Accordingly, this research formulates several flowshop scheduling problems with position-dependent learning and group effects. In particular, the learning effect of each job on every machine is based not only on its job position but also on its group position. Four objectives, namely, minimizing the makespan, total completion time, total weighted completion time, and maximum lateness, are considered. This research also shows the tight worst case ratios for several heuristics of the respective problems and derives the lower bound estimates to examine the performance of the proposed heuristics and meta-heuristics (genetic algorithm and quantum differential evolutionary algorithm). Finally, this research presents the result of the computational experiments, provides a case study on satellite production, and outlines some future research directions.

**1. Introduction**

The concept of learning curves was introduced almost 80 years ago (Wright, 1936), during which the research of machine scheduling problems practically began. Nevertheless, the learning effect in scheduling problems was first explored only a decade ago (Biskup, 1999). Since then, considerable efforts have been devoted to the study on scheduling with learning effect (Koulamas & Kyparisis, 2007; Mosheiov, 2001; Mosheiov & Sidney, 2003), which remains to be an extremely well-studied research area until today (Biskup, 2008; Jiang, Chen, & Kang, 2013; Janiak, Janiak, Krysiak, & Kwiatkowski, 2014; Qian & Steiner, 2013). The majority of the works in this field have focused on single-machine problem (and quite a few on the two-machine problem); yet, some works have recently explored the many variations of such problem.

A permutation flowshop group scheduling problem with position-based learning effect is considered in this study, which is motivated by existing literature and labor-intensive industrial applications such as various assembling processes. An industrial application is derived from a standard satellite components plant, in which the standard components are used to construct the body of a satellite. All the standard components prepared with the same processes (e.g., machining and assembling) are classified into several groups based on the similarity coefficient method to improve their productivity. Given that the components are largely produced with manual assembling processes in a flowshop manner, the learning effect cannot be ignored when the production schedules of these components are considered.

Below is a brief literature review that primarily focuses on flowshop and group scheduling with learning effect.

Lee and Wu (2004) were among the first scholars to explore the problem of flowshop variations (without considering group technology) in a two-machine flowshop case with a learning effect. In such a case, Wang (2005) revealed that the classic Johnsons rule (optimal algorithm for makespan minimization) is not optimal, denoting a huge difference between flowshop scheduling problems with learning effect and those without (original ones). Wang and Xia (2005) formulated the general flowshop problems with learning effect (i.e., for multiple machines) and provided a heuristic algorithm with worst-case bound. Meanwhile, Biskup (2008) exhaustively reviewed scheduling with learning effect grows by categorizing the learning effect scheduling problems into position-based and sum-of-processing-time based learning effect models. These learning effects (position-based and...
This research provides significant contributions to the field of flowshop scheduling. To the best of our knowledge, this is the first study on group scheduling with learning effects in a flowshop setting, in which a worst-case analysis for four given heuristics is conducted. In particular, this research presents two polynomially-solvable cases of the problem, develops two well-known evolutionary meta-heuristics to further improve the quality of the solutions obtained from the heuristics, and derives the lower bounds of the problem for the four objective functions to evaluate the quality of the solutions obtained from the meta-heuristics.

The remainder of the paper is organized into nine sections. Following the Introduction, Section 2 presents the problem statement. Section 3 proposes several heuristic algorithms with worst-case analysis for the given objective functions. Section 4 introduces two polynomially-solvable cases. Section 5 develops two well-known evolutionary algorithms to further improve the quality of the solutions obtained from the heuristics. Section 6 demonstrates the lower bound estimates for the problem. Section 7 reports the computational results and thoroughly discusses the group effect. Section 8 concentrates on the industrial application of the flowshop group scheduling problem, and Section 9 demonstrates the drawn conclusions and outlines the future research.

2. Problem statement

The flowshop group scheduling problem with learning effect is expressed as $\text{Pm|p_m||C_{\text{max}}}$ with the three-field notation of Graham, Lawler, Lenstra, and Kan (1979). The problem states that a set of $N$ jobs are formally grouped into $G$ groups and should be processed once on each of $M$ machines without preemption. Job $j$, $j = 1, 2, \ldots, N$, has a nonnegative normal processing time $p_{ijk}$ in group $i$, $i = 1, 2, \ldots, G$, on machine $k$, $k = 1, 2, \ldots, M$. In this case, permutation schedule is considered, and the intermediate storage between the successive machines is unlimited. A schedule includes both the group and job schedules within each group (i.e., jobs within one group can only be scheduled as a “block”). Four minimization objectives (i.e., makespan, total completion time, total weighted completion time, and maximum lateness) are also regarded in such a case.

Below is a list of the notations used throughout the paper. To be specific, we denote $n_i$ as the number of jobs in group $i$, such that $\sum_{i=1}^{G} n_i = N$, and denote the maximum of which to be $n_{\text{max}}$. We follow the classic notation of defining completion time of job $j$ in group $i$ as $C_{ij}$, and $C_{\text{max}}$ if we specialize in the completion time on machine $k$. Furthermore, we associate each job with a weight, denoted $\omega_j$, which is the cost rate for delaying its completion. $d_j$ is the due date for job $j$ in group $i$, before which the job should be completed. In the problem, we assume the learning effect for each group and each job is only dependent on its location in the job sequence, i.e., we denote the normal processing time (without considering the learning effect) of job $j$ in group $i$ on machine $k$ as $p_{ijk}$ then the actual processing time of a job in group position $i$ and job position $j$ when operated by machine $k$ is $p_{ijk}^{(m)} (\alpha_1, \alpha_2 < 0)$. Square brackets are used to denote that $i$ and $j$ indicate the $i$th and $j$th positions, respectively, which are different from the primal coding of the jobs. This notation is consistently used throughout the paper.

In contrast to the normal setup times defined as $s_{ij}$ (for group $i$ on machine $k$), the actual setup time of a group in position $i$ is given by $s_{ij}^{(m)}$, which implies that the setup time decreases when time goes on and when workers are more skilled for the job. In this case, the $i$th group (with a single setup time) is assumed impotent to begin processing jobs on the first machine until $s_{ij}^{(m)}$ after the