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A computational comparison of formulations for the economic lot-sizing with remanufacturing

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ABSTRACT

An important way to try reducing environmental damage in the manufacture of industrialized goods is through the use of production systems which deal with the reuse of returned materials such as reverse logistics. In this paper, we consider a production planning problem arising in the context of reverse logistics, namely the economic lot-sizing with remanufacturing (ELSR). In the ELSR, deterministic demand for a single item over a finite time horizon has to be satisfied, which can be performed from either newly produced or remanufactured items, and the goal consists in minimizing the total production costs. Our objective is to devise approaches to solve larger (more difficult) instances of the problem available in the literature to optimality using a standard mixed-integer programming (MIP) solver. We present a multicommodity extended formulation and a strengthened Wagner–Whitin based formulation, which makes use of *a priori* addition of newly described valid inequalities in the space of original variables. We also propose a novel dynamic heuristic measure based on the cost structure to automatically determine the size of a partial version of the Wagner–Whitin based formulation. Computational results show that the novel partial Wagner–Whitin based formulation with the size automatically determined in a heuristic way outperforms all the other tested approaches, including a best performing shortest path formulation available in the literature, when we consider the number of instances solved to proven optimality using a standard MIP solver. This new approach allowed to solve to optimality more than 96% of the tested instances for the ELSR with separate setups, including several instances that could not be solved otherwise.

1. Introduction

The economic lot-sizing with remanufacturing (ELSR) has received great attention in recent years, and one of the reasons is the increasing interest in the search for better ways to provide sustainable production systems that can be implemented effectively. The problem consists in planning the production of new items from raw materials together with the remanufacture of returned items in order to satisfy the deterministic demands over a finite discrete time horizon while minimizing the total production costs. The problem was independently shown to be NP-Hard in Baki, Chaouch, and Abdul-Kader (2014) and Retel Helmrich, Jans, van den Heuvel, and Wagelmans (2014) (also Retel Helmrich (2013)).

A basic production planning problem in the literature is the uncapacitated lot-sizing (ULS). Wagner and Whitin (1958) considered the ULS in a seminal work on the algorithmic treatment given to production planning problems. Barany, Van Roy, and Wolsey (1984a,b) proposed the (l, S)-inequalities and showed that together with basic constraints they give the convex hull of the set of feasible solutions. Extended formulations were proposed in Krarup, Bilde, and location (1977) (multicommodity or facility location) and Eppen and Martin (1987) (shortest path). Since then, valid inequalities have been widely used to treat several production planning models (see Pochet & Wolsey (2006) for an extensive review).

The economic lot-sizing with remanufacturing is an extension of the ULS in which remanufacturing options are available and has been recently studied in several works. Richter and Sombrutzki (2000) treated a simple version of the problem in which both production and remanufacture are unlimited, i.e. the amount of returned items available at the beginning of the planning horizon is enough to satisfy the entire demand, and the costs are time invariant. The authors analyzed the properties of optimal

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solutions and proposed a dynamic programming algorithm. Richter and Weber (2001) extended this uncapacitated model to treat time variant costs in a reverse Wagner–Whitin model with variable manufacturing and remanufacturing costs.

Teunter, Bayindir, and van den Heuvel (2006) treated the economic lot-sizing with remanufacturing with both separate and joint setup costs. The authors obtained a polynomial time algorithm for the case with joint setup and stationary costs and proposed heuristics for both joint and separate setup variants. Later, Schulz (2011) generalized the Silver-Meal heuristic presented in (Teunter et al., 2006). Baki et al. (2014) proposed an alternative mixed-integer programming (MIP) formulation for the problem and showed that it provided better linear relaxation bounds than a standard formulation. They also developed a dynamic programming based heuristic and performed extensive numerical experiments, using several instances with a small planning horizon of 12 periods and some with larger planning horizons, to validate the good performance of the heuristic.

Retel Helmrich et al. (2014) compared MIP approaches to the ELSR and proposed a shortest path formulation, an approximate shortest path formulation and valid inequalities based on the (l, S, WW)-inequalities for the ULS which were added *a priori* to a standard formulation in order to obtain a Wagner–Whitin based formulation. They performed computational experiments comparing the approaches and also showed that the problem with joint setups is NP-Hard when the costs are time variant. Recently, Sifaleras, Konstantaras, and Mladenović (2015) developed a variable neighborhood search heuristic (VNS) to the problem. The proposed VNS heuristic outperformed the state-of-the-art heuristic methods from the literature in the reported computational experiments using a set of benchmark instances (6480 instances with 12 periods each) proposed by Schulz (2011). They also presented a new benchmark set of larger (more difficult) instances, with 52 periods, and demonstrated the robustness of the approach using these new instances. Some authors also considered multi-item extensions of the ELSR. Sahling (2013) proposed a column generation approach for a multi-item extension of the ELSR which also included big bucket capacity constraints on production and remanufacture. More recently, Sifaleras and Konstantaras (in press) studied another multi-item variant of the problem and proposed a variable neighborhood descent (VND) heuristic which was shown to outperform the use of a standard MIP solver through computational experiments.

Our work concentrates on mixed-integer programming approaches in an attempt to, using a standard MIP solver, solve to optimality the largest instances of the economic lot-sizing with remanufacturing available in the literature. Therefore, we limited ourselves to two benchmark sets of instances: the first one proposed by Sifaleras et al. (2015) (108 instances with 52 periods each), and the second proposed by Retel Helmrich et al. (2014) (120 instances with 25 periods, 120 instances with 50 periods and 120 instances with 75 periods).

The economic lot-sizing with remanufacturing can be formally defined as follows. There is a single item with deterministic demand over a finite discrete time horizon of NT periods. The demand for each period $t \in \{1 \ldots NT\}$ is d_t and the amount of returned items arriving at each period is r_t. Production of new items is unlimited while the remanufacture is restricted to the availability of returned items. There are fixed and variable production costs (respectively f_t^p and $\bar{p}_t^p$) as well as fixed and variable remanufacture costs (respectively f_t^r and $\bar{p}_t^r$). The storage of finished items implies a cost of h_t^p per unit and that of returned items a cost of h_t^r per unit. The ELSR has two main variants, namely the economic lot-sizing with remanufacturing and separate setups (ELSRs) in which there are separate setups for production and for remanufacture, and the economic lot-sizing with remanufacturing and joint setups (ELSRj) in which both production and remanufacture share the same setup. It is assumed that there is no initial stock of either finished or returned items and no final stocks of finished items. In addition, all data is non negative, the cumulated demand in the interval $[k, t]$ is defined as $d_{kt} = \sum_{l=k}^{t} d_l$ for $1 \leqslant k \leqslant t \leqslant NT$ and the cumulated amount of returned items in the interval $[k, t]$ as $r_{kt} = \sum_{l=k}^{t} r_l$ for $1 \leqslant k \leqslant t \leqslant NT$.

The remainder of the paper is organized as follows. In Section 2 we formally define the economic lot-sizing with remanufacturing and separate setups using a standard MIP formulation. The shortest path formulation of Retel Helmrich et al. (2014) is presented in Section 3. A new multicommodity formulation is introduced in Section 4, and a Wagner–Whitin based formulation is given in Section 5 together with new valid inequalities to the problem and a heuristic technique to determine the size of a partial formulation based on the problem's cost structure. In Section 6 we show how the approaches devised for ELSRs can be adapted to deal with ELSRj. Computational experiments are summarized in Section 7. The results show that the multicommodity formulation outperforms the considered shortest path formulation (which was the best approach in (Retel Helmrich et al., 2014) when we consider the number of instances solved to optimality) for most of the cases, and that the partial Wagner–Whitin formulation with automatically determined size outperforms all the other approaches allowing to solve several additional instances to optimality. Some final remarks are discussed in Section 8.

2. The economic lot-sizing with remanufacturing and separate setups

In this section, we present a formal description of the economic lot-sizing with remanufacturing and separate setups (ELSRs) using a standard mixed-integer programming formulation.

With the purpose of formulating the problem as a mixed-integer program, consider x_t^p to be the amount of items produced in period t, x_t^r to be the amount of items remanufactured in period t, s_t^p to be the amount of finished items in stock at the end of period t, s_t^r to be the amount of returned items in stock at the end of period t, y_t^p to be equal to 1 if production happens in period t and to be 0 otherwise, and y_t^r to be equal to 1 if remanufacture happens in period t and to be 0 otherwise. Using the variables just described, the problem can be formulated as

$$\min \sum_{t=1}^{NT} (h_t^p s_t^p + \bar{p}_t^p x_t^p + f_t^p y_t^p) + \sum_{t=1}^{NT} (h_t^r s_t^r + \bar{p}_t^r x_t^r + f_t^r y_t^r) \tag{1}$$

$$s_{t-1}^p + x_t^p + x_t^r = d_t + s_t^p, \quad \text{for } 1 \leqslant t \leqslant NT, \tag{2}$$

$$s_{t-1}^r + r_t = x_t^r + s_t^r, \quad \text{for } 1 \leqslant t \leqslant NT, \tag{3}$$

$$x_t^p \leqslant d_{t,NT} y_t^p, \quad \text{for } 1 \leqslant t \leqslant NT, \tag{4}$$

$$x_t^r \leqslant \min\{r_{1t},\ d_{t,NT}\} y_t^r, \quad \text{for } 1 \leqslant t \leqslant NT, \tag{5}$$

$$x^p,\ x^r,\ s^p,\ s^r \in \mathbb{R}_+^{NT}, \tag{6}$$

$$y^p,\ y^r \in \{0, 1\}^{NT}. \tag{7}$$

The objective function minimizes the total cost. Constraints (2) are balance constraints regarding the finished items while constraints (3) are balance constraints for the returned items. Constraints (4) force the production setup variables to be equal to one if production occurs. Constraints (5) enforce the remanufacture setup variables to be equal to one if remanufacture occurs. Note that, differently from Retel Helmrich et al. (2014), in (5) the cumulated returns r_{1t} are also considered as upper bounds on remanufacture in an attempt to obtain better linear relaxation