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Robust optimization under correlated polyhedral uncertainty set ☆

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ABSTRACT

Uncertain data in practical optimization problems led to emerge of robust optimization approaches, whereby solutions with more stable quality against perturbations are constructed. Furthermore, to avoid over-conservatism, different kinds of uncertainty sets are introduced. In most of these approaches, uncertain coefficients of the problems are assumed to be independent. While in practice, these coefficients are often influenced by several common uncertainty sources which cause dependency among uncertain coefficients. In this research, a new uncertainty set based on estimated correlation matrix of uncertain coefficients is introduced. It is followed by a robust counterpart formulation of the problem using the proposed uncertainty set. To evaluate the performance of the proposed model it is applied on a couple of uncertain optimization problems. The experimental results revealed that when significant correlations between the coefficients exist, the performance of the proposed method is superior to that of the traditional polyhedral uncertainty set. The results are discussed and concluding remarks are made.

1. Introduction

In a real-world environment where the values of the corresponding parameters are influenced by unpredictable events, dealing with uncertainties are inevitable. In business applications, social sciences, physical sciences and engineering, data are often incomplete or contain measurement errors (Mulvey, Vanderbei, & Zenios, 1995). Ben-Tal, El Ghaoui, and Nemirovski (2009) demonstrated that in a linear programming problem, a small perturbation in nominal data may result in severely infeasibility of optimal solution. There are several approaches including stochastic optimizations, robust optimizations and fuzzy programming that can be used to model uncertainty (Soltani, Sadjadi, & Tavakkoli-Moghaddam, 2013). In the stochastic optimization approach, it is assumed that the uncertain data is random and its probability distribution is known (Ben-Tal et al., 2009). Deciding about the appropriate probability density function for a set of historical data may be difficult as it contains inaccuracy. Furthermore, sometimes lack of historical data makes it impractical to determine a probabilistic distribution of the uncertainty. Robust optimization approach formulates the uncertainty assuming that the value of an uncertain coefficient varies in a known interval rather than proposing a probability distribution.

Soyster's approach in formulating robust models is one of the earliest works in this area (Soyster, 1973). He simply assumed that for an uncertain optimization problem each coefficient may perturb independently throughout an interval. In such an environment the robust solution can be obtained by taking into account the worst value of each coefficient. Despite the fact that the Soyster's approach guaranties feasible solutions with respect to every possible realization of coefficients, it results over-conservatism.

To avoid over-conservatism, Ben-Tal and Nemirovski (1998), EI-Ghaoui and Lebret (1997), El Ghaoui, Oustry, and Lebret (1998) in independent works, developed robust models by making a tradeoff between the robustness and optimal value of the objective function. They proposed an ellipsoidal uncertainty set upon which they introduced robust counterpart formulation. As another prominent studies to avoid over-conservatism the works of Bertsimas and Sim (2003, 2004) are considerable. They proposed a polyhedral uncertainty set and the corresponding robust counterpart which its level of robustness can be adjusted. Moreover, their model has the advantage of lower computational burden because of the linear structure compared with the former robust model under ellipsoidal set. This enabled the researchers to widely use the model in different areas includes logistics and production systems (Hazır & Dolgui, 2013; Lu, Ying, & Lin, 2014), power engineering (Baringo & Conejo, 2011; Thatte, Viassolo, & Xie, 2012), portfolio selection problem (Gregory, Darby-Dowman, & Mitra, 2011), data envelopment analysis (Omrani, 2013; Shokouhi, Hatami-Marbini, Tavana, & Saati, 2010), etc. The ellipsoidal and

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polyhedral uncertainty sets are introduced more in detail in Sections 2.2 and 2.3, respectively.

The research carried out by Mulvey et al. (1995) can also be considered as another valued research for an environment with uncertain coefficients with finite discrete values. They developed a robust model where the value of the uncertain coefficients are described using a set of scenarios each one with a specific probability. The objective function of the corresponding model is determined considering optimality robustness as well as the feasibility robustness. Their approach yields a series of solutions that are less sensitive to realizations of the data in a set of scenarios (Rahmani, Ramezanian, Fattahi, & Heydari, 2013).

Besides tackling with uncertainty sets, determining an appropriate interval for each uncertain coefficient is another prominent issue for robust formulation. When historical data exist they can be used to determine the bounds of the interval. For this purpose, usually the minimum and the maximum observed data are used as the lower and the upper bounds, respectively. In addition Pachamanova (2002) argued that if information about the standard deviation of the uncertain coefficients is available, it can be employed to determine the bounds of intervals. This proposed method uses the normalized value of the ratio of perturbation to the standard deviation of the historical data to model uncertainties. Also, Bertsimas and Pachamanova (2008) suggested a robust polyhedral formulation model based on the above addressed approach for a multi-period portfolio selection problem where future asset returns were considered as uncertain coefficients.

Bertsimas and Sim (2004) stated when correlation between data exists, it is required to determine the interval based on the sources of uncertainties. They assumed that the actual value of a specific coefficient depends on the estimated value plus the variations caused by uncertain sources. The proposed approach is formulated as follows:

$$\tilde{a}_{ij} = a_{ij} + \sum_{k\in K_i} \tilde{\eta}_{ik} g_{kj} \tag{1}$$

where a_{ij} and $\tilde{a}_{ij}$ represent the nominal and the actual value of uncertain coefficient, respectively. g_{kj} is a value which indicates how the uncertainty source k acts on $\tilde{a}_{ij}$, and $\tilde{\eta}_{ik}$ denotes the value of independent and symmetrically distributed random variables varying in a range of $[-1, 1]$. For details the readers are referred to (Bertsimas & Sim, 2004). The review of the research reveals this approach is used as a prevailing approaches to determine the bounds of the interval of uncertain coefficients. In this approach, the effectiveness of the model depends on how the values of a_{ij} and g_{kj} are determined. Ferreira, Barroso, and Carvalho (2012) employed Bertsimas and Sim's approach for correlated data in a demand response model and offered a couple of procedures to determine these values based on Principle Component Analysis (PCA) and Minimum Power Decomposition (MPD) whereby K principle sources of uncertainties are identified from H-variate data. This is followed by generating the value of the corresponding g_{kj}.

In general, when the sources of uncertainties are known and the corresponding data can be identified, the approach proposed by Bertsimas and Sim is an effective approach to obtain robust solutions. However, in practice it is usually difficult to identify such sources. For instance, in a production optimization problem the prices of the raw materials could be subject to uncertainty. This uncertainty can be caused by several sources such as the price of complementary or substitute goods, transportation costs, tax rate, etc. Each of these sources can have a substantial effect on the prices of raw materials but clearly defining all these sources is very difficult. Furthermore, even if the sources are known, it is more complicated to determine an appropriate function between the uncertain coefficients and the source. In such a circumstance, an alternative approach can be developed to formulate the robust optimization model considering the correlation between uncertain coefficients without necessity to identify the detailed sources.

In this study, we consider a linear programming problem with uncertain coefficients and assume that the estimate of the correlation matrix between uncertain coefficients is available, but recognizing the sources of uncertainty is not possible. We introduce a new polyhedral uncertainty in which its domain depends on the values of the correlation matrix. This matrix is assumed to be exploited from the historical data. Then, a robust counterpart is formulated based on the introduced uncertainty set. Furthermore, the performance of the proposed robust optimization model is studied.

The rest of this paper is organized as follows: In Section 2, a summary of the most prominent uncertainty sets and corresponding robust counterpart problems are reviewed. In Section 3, the new uncertainty set is introduced. In Section 4, the mathematical formulation of the proposed uncertainty set is presented. The model of the robust counterpart optimization problem under the introduced uncertainty set is introduced in Section 5. In Section 6, the experimental results are presented and finally in Section 7, the results are discussed and concluding remarks are made.

2. Review of literature on uncertainty sets and respective robust optimization models

In this section, the most prominent uncertainty sets and corresponding robust optimization models proposed by the researchers are briefly reviewed. To this aim, let consider the following uncertain linear programming problem under different uncertainty sets:

$$\begin{aligned} \max\ & \boldsymbol{c}'\boldsymbol{X} \\ & \boldsymbol{AX} \leqslant \boldsymbol{b} \\ & \boldsymbol{l} \leqslant \boldsymbol{X} \leqslant \boldsymbol{u} \end{aligned} \tag{2}$$

Without loss of generality it is often assumed that only matrix **A** is subject to uncertainty. If the coefficients of the objective function or right hand side of the constraints are subject to uncertainty, it can be formulated as follows (Li, Ding, & Floudas, 2011):

$$\begin{aligned} & max && z \\ & Subject\ to: && z - \boldsymbol{c}'\boldsymbol{x} \leqslant 0 \\ & && x_0\boldsymbol{b} + \boldsymbol{Ax} \leqslant 0 \\ & && x_0 = -1 \\ & && \boldsymbol{l} \leqslant \boldsymbol{x} \leqslant \boldsymbol{u} \end{aligned} \tag{3}$$

The uncertain matrix **A** is composed of entries $\tilde{a}_{ij}$ as actual values of coefficients and can take values in the range $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}]$ where a_{ij} is the nominal value of $\tilde{a}_{ij}$, and $\hat{a}_{ij}$ detes the maximum positive perturbation. Hence $\tilde{a}_{ij}$. can be defined as:

$$\tilde{a}_{ij} = a_{ij} + \zeta_{ij}\hat{a}_{ij} \qquad -1 \leqslant \zeta_{ij} \leqslant 1 \tag{4}$$

where in general, ζ_{ij} is a random variable which is subject to uncertainty and perturbs in the range $[-1, 1]$. In the remaining of this section the uncertain problem (2) is considered under different uncertainty sets.

2.1. Box uncertainty set and its robust formulation

In an uncertain optimization problem, it is assumed that ζ_{ij}s are random variables and independent. Their absolute values vary between zero and Ψ_i. The interaction of perturbations creates a box which is called Box Uncertainty. This uncertainty set can be described as follows:

$$U^A = \{\tilde{a}_{ij} = a_{ij} + \zeta_{ij}\hat{a}_{ij} \,|\, |\zeta_{ij}| \leqslant \Psi_i, \forall i\} \tag{5}$$