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Exact algorithms for single-machine scheduling problems with a variable maintenance

Kuo-Ching Ying [a], Chung-Cheng Lu [b,*], Jhao-Cheng Chen [a]

[a] Department of Industrial Engineering and Management, National Taipei University of Technology, Taipei, Taiwan
[b] Department of Transportation and Logistics Management, National Chiao-Tung University, Hsinchu, Taiwan

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ABSTRACT

This paper deals with four single-machine scheduling problems (SMSPs) with a variable machine maintenance. The objectives of the four SMSPs are to minimize mean lateness, maximum tardiness, total flow time and mean tardiness, respectively. These four SMSPs are important in the literature and in practice. This study proposes an exact algorithm with the computational complexity $O(n^2)$ for each of the four SMSPs. In addition to the given jobs, the machine maintenance activity between two consecutive jobs is optimally scheduled.

1. Introduction

Single-machine scheduling problems (SMSPs) are quite common in manufacturing systems, and the related topics of scheduling theories or real applications have been studied for several decades (e.g., Billaut, Della Croce, & Grosso, 2015; Huang & Yang, 2013; Lin, Chou, & Ying, 2007; Lin & Ying, 2013; Lu, Lin, & Ying, 2012; Oron, Shabtay, & Steiner, 2015). Comprehensive surveys and reviews of SMSPs have been conducted in the literature (e.g., Abdul-Razaq, Potts, & Van Wassenhove, 1990; Pinedo, 2012). In the real world, machine maintenance is one of the critical factors that can significantly affect job scheduling performance. Since the 1990 s, scheduling problems resulting from machine maintenance issues have received considerable attention. However, most studies in the literature have assumed the two critical factors of machine maintenance, i.e., starting time and duration, to be deterministic and fixed (e.g., Epstein et al., 2012; Ma, Chu, & Zuo, 2010; Mati, 2010; Moncel, Thiery, & Waserhole, 2014; Mor & Mosheiov, 2012).

Kubzin and Strusevich (2006) were among the first to introduce the concept of variable machine maintenance to machine scheduling problems. They considered two-machine scheduling problems and treated the starting time and the duration of the maintenance operation as decision variables. They showed the resulting scheduling problems to be polynomially solvable and proposed a fully polynomial-time approximation scheme and a fast 3/2-approximation algorithm. Ji, He, and Chen (2007) studied an SMSP with periodic maintenance activities, the objective being to find a schedule that minimizes the makespan, subject to periodic maintenance and nonresumable jobs. They showed that there is no polynomial-time approximation algorithm with a worst-case ratio of less than 2 unless P = NP. Yang, Yang, and Cheng (2010) showed that a single-machine due-window assignment and scheduling problem with job-dependent aging effects and deteriorating maintenance can be optimally solved in $O(n^4)$ time. In the same year, Xu, Yin, and Li (2010) considered SMSPs and parallel-machine scheduling problems, with a requirement that the duration of performing one maintenance activity is an increasing linear function of the total processing time of all jobs. In their study, two approximation approaches were developed to solve the scheduling problems in order to minimize the makespan. Mosheiov and Sidney (2010) considered an optional maintenance in SMSPs, assuming the maintenance duration to be a non-decreasing function of its starting time. Their study showed that SMSPs which minimize the makespan, flow time, maximum lateness, total earliness, tardiness and due-date costs and number of tardy jobs are polynomially solvable. Luo, Chen, and Zhang (2010) addressed the SMSP and considered a maintenance activity with a starting time that does not exceed its due date and with duration that is a non-decreasing function of its starting time. In their study, two approximation algorithms were developed to solve the problem which minimizes the total weighted completion time.

Cheng, Hsu, and Yang (2011) investigated the unrelated parallel-machine scheduling problem with deteriorating maintenances which minimizes the total completion time or the total

* Corresponding author.
E-mail address: jasoncclu@gmail.com (C.-C. Lu).

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machine load. In this problem, each machine has at most one maintenance activity, which can be performed at any time, and the duration of each maintenance activity is assumed to increase linearly with its starting time. Their study showed that both versions of the problem can be optimally solved in polynomial time. Yang, Cheng, Yang, and Hsu (2012) proposed an efficient algorithm to solve the unrelated parallel-machine scheduling problem with aging effects and multi-maintenance activities simultaneously. Their objective was to minimize the total machine load when the maintenance frequencies on the machines are given. Cheng, Yang, and Yang (2012) investigated an SMSP of the common due-window assignment and scheduling of linear time-dependent deteriorating jobs and a deteriorating maintenance activity in order to simultaneously minimize the earliness, tardiness, due-window starting time and due-window size costs. The authors presented polynomial-time solution algorithms for the problem and some of its special cases. Yang (2012) studied SMSPs with multi-maintenance activities and learning effects, assuming that the duration of each maintenance activity depends on the running time of the machine. The objectives of the two problems were to determine the optimal maintenance frequencies, maintenance positions and schedule of all jobs so as to minimize the makespan and total completion time, respectively. The study further showed the SMSPs addressed to be solvable in polynomial time.

Recently, Yin, Wu, Cheng, and Wu (2014) provided polynomial-time solution algorithms for various versions of SMSPs with simultaneous consideration of the due-date assignment, generalized position-dependent deteriorating jobs and deteriorating maintenance activities. Their objective was to jointly determine the optimal job sequence, maintenance frequency and maintenance positions so as to minimize the cost of the due-date assignment, the cost of discarding jobs that cannot be completed by their due dates and the earliness of the scheduled jobs. Luo, Cheng, and Ji (2015) followed by addressing four SMSPs, each having a maintenance activity whose duration increases with its starting time. They presented polynomial-time algorithms for the four problems aimed at minimizing the makespan, sum of completion time, maximum lateness and number of tardy jobs, respectively. In particular, they showed that the algorithms found exact solutions based on the dispatching rules of the shortest processing time (SPT) and earliest due date (EDD) as categorized by Blackstone, Phillips, and Hogg (1982). More recently, Yin, Xu, Cheng, Wu, and Wang (2016) considered an SMSP with independent and simultaneously available jobs without preemption, where the machine has a fixed maintenance activity. The authors proposed two pseudo-polynomial dynamic programming algorithms and a fully polynomial-time approximation scheme to minimize the total amount of late work.

Along the lines of Luo et al. (2015), the aim in this research was to investigate another four SMSPs with the objective of minimizing mean lateness, maximum tardiness, total flow time and mean tardiness, respectively. These four SMSPs have received much attention in the literature (Cheng, Hsu, Huang, & Lee, 2011; Herr & Goel, 2016; Huang, Yu, & Yang, 2013; Kacem & Chu, 2008; Lin, Lu, & Ying, 2011; Mazdeh, Rostami, & Namaki, 2013; Xiong, Xing, & Wang, 2015; Yin, Ye, & Zhang, 2014).

The steel strip production in a steel plant can be considered as a practical example for the proposed SMSPs (Luo et al., 2015). During the production process, steel slabs (jobs) must pass a re-heat furnace (machine) before they are rolled into strips. The re-heat furnace must be maintained, i.e., cleaned and fuels refilled, prior to a given deadline to ensure that it functions normally. This action can be regarded as a “variable maintenance activity” because the duration of cleaning and refilling the furnace is a positive and non-decreasing function of the total processing time of the steel slabs that have been processed. For solving these SMSPs, in this paper we propose an optimal algorithm with the computational complexity $O(n^2)$ for each of them.

The rest of this paper is organized as follows. The following section defines the problems under consideration. Section 3 presents the polynomial-time algorithms for solving the four SMSPs and the respective numerical examples. Section 4 gives the conclusions of this study.

2. Problem definition

Consider a sequencing problem that has a set $J = \{1, \ldots, n\}$ of n independent jobs to be processed on a single-machine without interruption. Each job j $(j = 1, \ldots, n)$ has a processing time, p_j, and a due date, d_j. All jobs are simultaneously available for processing at the beginning of the planning horizon. The machine is continuously available but must undertake one maintenance activity during the planning horizon, where the starting time, s, of the maintenance activity must be before a given deadline, s_d, i.e., $s \leqslant s_d$. The starting time, s, of the maintenance activity is a decision variable which is determined by the scheduler, and the duration of maintenance, l, is a positive and non-decreasing function of its starting time, i.e., $l = f(s)$ and $f(s_b) \geqslant f(s_a)$ for all $s_b > s_a$. The aim is to optimally determine the starting time of the maintenance and the sequence of all jobs under various scheduling objectives.

Let C_j, r_j, and F_j, $j = 1, \ldots, n$, be the completion time, release time and flow time for job j, respectively. The mean lateness is denoted by $\bar{L}$, and measured by the difference between the unequal processing times and the due dates of the jobs. The mean tardiness and maximum tardiness are denoted by $\bar{T}$ and $T_{\max}$, respectively, and measured based on the parts of the processing times that exceed the due dates of the jobs. Using the three-field classification method of Graham, Lawler, Lenstra, and Kan (1979), the four SMSPs concerned with minimizing the mean lateness, maximum tardiness, total flow time and mean tardiness can be represented as $1, VM||\bar{L}$, $1, VM||T_{\max}$, $1, VM|r_j = r|\sum_j F_j$ and $1, VM|d_j = d|\bar{T}$, respectively, where VM denotes the variable maintenance. Note that each problem considers the requirement $s_d < \sum_{j=1}^{n} p_j$, and to ensure that the maintenance is not arranged after the last job was finished, because that position results in optimal position of maintenance activity, and makes the problems equivalent to the ones without any maintenance activity. For the ease of illustrating the scheduling processes, we further let $J_{[i]}$, $i = 1, \ldots, n$, be the job at the ith processing position, and π^i be the candidate schedule with the maintenance activity arranged before the start time of $J_{[i]}$; that is, the maintenance activity executed at time zero when $i = 1$.

Let x_{ij} $(i = 1, \ldots, n; j = 1, \ldots, n)$ be the binary variables. x_{ij} is equal to 1 if job j is assigned to position i; otherwise, $x_{ij} = 0$. Then, the $1, VM||\bar{L}$, $1, VM||T_{\max}$, $1, VM|r_j = r|\sum_j F_j$ and $1, VM|d_j = d|\bar{T}$ problems can be formulated as mixed integer linear programming (MILP) mathematical models, as presented in the following subsections, respectively.

2.1. MILP model of the $1, VM||\bar{L}$ problem

Minimize z
subject to

$$z \geqslant \sum_{k=1}^{n} (C_{[k]}(\pi^i) - d_{[k]})/n, \quad i = 1, 2, \ldots, n, \tag{1}$$

$$C_{[k]}(\pi^i) = \sum_{i=1}^{k} \sum_{j=1}^{n} x_{ij} p_j, \quad \forall k = 1, \ldots, i-1, \tag{2}$$