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Short Communication

On "A fuzzy bi-criteria transportation problem": A revised algorithm

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ABSTRACT

Keshavarz and Khorram formulated a fuzzy bi-criteria transportation problem with fuzzy delivery time and fuzzy profit of transportation, as two conflicting objectives (Keshavarz & Khorram, 2011). They used the max–min criterion of Bellman and Zadeh to reformulate the presented fuzzy bi-criteria transportation problem as a single objective non-linear programming problem, then showed that the optimal solution of this non-linear programming can be found by solving a bi-level programming problem. Finally, they proposed an algorithm based on the parametric linear programming for solving this bi-level problem. In this paper, a shortcoming of Keshavarz and Khorram's algorithm is pointed out and a revised algorithm is proposed to solve the problem. In order to illustrate the performance of this algorithm, Keshavarz and Khorram's example is used and its optimal solution is improved.

1. Introduction

Keshavarz and Khorram (2011) introduced and formulated a Fuzzy Bi-Criteria Transportation Problem (FBCTP), and reformulated their presented FBCTP as a crisp single objective non-linear programming problem, using the Bellman–Zadeh's fuzzy max–min criterion (Bellman & Zadeh, 1970). They found optimality conditions of solution and showed that the optimal solution of this non-linear programming can be obtained by solving a bi-level programming problem, which its lower-level is a bi-objective problem. Finally they proposed an algorithm, based on the parametric programming, for solving this bi-level problem and designed a comparative analysis to find the optimal solution of this non-linear programming.

In this paper a shortcoming of Keshavarz and Khorram's algorithm is pointed out and a revised algorithm is presented to obviate this shortcoming; finally through their numerical example, the applicability of this algorithm will be demonstrated.

2. Keshavarz and Khorram's FBCTP formulation

Keshavarz and Khorram (2011) formulated the following FBCTP.

$$\begin{aligned}
\min \quad & T(\boldsymbol{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} t_{ij}x_{ij} \\
\max \quad & P(\boldsymbol{x}) = \sum_{i=1}^{m}\sum_{j=1}^{n} p_{ij}x_{ij} \\
\text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = S_i \qquad i = 1,\ldots,m \\
& \sum_{i=1}^{n} x_{ij} = D_j \qquad j = 1,\ldots,n \\
& x_{ij} \geqslant 0 \qquad i = 1,\ldots,m; \quad j = 1,\ldots,n.
\end{aligned} \tag{1}$$

where t_{ij} and p_{ij} are fuzzy variables associated with fuzzy delivery time $\tilde{t}_{ij} = \langle \alpha_{ij}, \beta_{ij} \rangle$ and fuzzy profit $\tilde{p}_{ij} = \langle a_{ij}, b_{ij} \rangle$ on link (i,j), respectively; their membership functions are defined by (2) and (3). x_{ij}, as a decision variable, is the number of units shipped along link (i,j) from origin i to destination j. $S_i > 0, i = 1,\ldots,m$, and $D_j > 0, j = 1,\ldots,n$, denote units of a particular item (commodity) are supplied by source node i, and units are required by destination node j, respectively. Furthermore, assume that the problem is balanced, i.e. $\sum_{j=1}^{n} S_i = \sum_{i=1}^{m} D_j$.

$$\mu_{ij}(t_{ij}) = \begin{cases} 1 & t_{ij} \geqslant \beta_{ij}, x_{ij} > 0 \\ \frac{t_{ij}-\alpha_{ij}}{\beta_{ij}-\alpha_{ij}} & \alpha_{ij} \leqslant t_{ij} \leqslant \beta_{ij}, x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

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$$\pi_{ij}(p_{ij}) = \begin{cases} 1 & p_{ij} \leqslant a_{ij}, x_{ij} > 0 \\ \frac{b_{ij}-p_{ij}}{b_{ij}-a_{ij}} & a_{ij} \leqslant p_{ij} \leqslant b_{ij}, x_{ij} > 0 \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

In order to solve the problem (1), Keshavarz and Khorram formulated the total delivery time and total profit of transporting commodities as the following fuzzy intervals, respectively.

$$\bar{\mu}(T(\boldsymbol{x})) = \begin{cases} 1 & T(\boldsymbol{x}) \leqslant \alpha \\ \frac{\beta - T(\boldsymbol{x})}{\beta-\alpha} & \alpha \leqslant T(\boldsymbol{x}) \leqslant \beta \quad \forall \boldsymbol{x} \in X \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

$$\bar{\pi}(P(\boldsymbol{x})) = \begin{cases} 1 & P(\boldsymbol{x}) \geqslant b \\ \frac{P(\boldsymbol{x})-a}{b-a} & a \leqslant P(\boldsymbol{x}) \leqslant b \quad \forall \boldsymbol{x} \in X \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

where X is the set of all feasible solutions of the problem (1), $\alpha = \min_{\boldsymbol{x}\in X}\sum_{i=1}^{m}\sum_{j=1}^{n}\alpha_{ij}x_{ij}$, $\beta = \max_{\boldsymbol{x}\in X}\sum_{i=1}^{m}\sum_{j=1}^{n}\beta_{ij}x_{ij}$, $a = \min_{\boldsymbol{x}\in X}\sum_{i=1}^{m}\sum_{j=1}^{n}a_{ij}x_{ij}$, and $b = \max_{\boldsymbol{x}\in X}\sum_{i=1}^{m}\sum_{j=1}^{n}b_{ij}x_{ij}$.

Keshavarz and Khorram applied the Bellman and Zadeh's max–min criterion to convert the FBCTP (1) to the following problem.

$$\max_{\boldsymbol{x}\in X}\left(\min_{\{(i,j)|x_{ij}>0\}}\left\{\mu_{ij}(t_{ij}), \bar{\mu}(T(\boldsymbol{x})), \pi_{ij}(p_{ij}), \bar{\pi}(P(\boldsymbol{x}))\right\}\right) \tag{6}$$

After some analytical and computational manipulation, Keshavarz and Khorram (2011) proved that the problem (6) can be transformed into the following bi-level programming problem.

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & f(\lambda,\boldsymbol{x}) \leqslant 0, \quad g(\lambda,\boldsymbol{x}) \leqslant 0, \\ & f(\lambda,\boldsymbol{x}) \cdot g(\lambda,\boldsymbol{x}) = 0 \\ & \boldsymbol{x} \in \overline{X} \end{aligned} \tag{7}$$

where $\overline{X} \subseteq X$ is the set of all efficient solutions of the following bi-objective problem, as the lower-level problem.

$$\begin{aligned} \min \quad & T(\boldsymbol{x},\lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n}(\alpha_{ij} + (\beta_{ij}-\alpha_{ij})\lambda)x_{ij} \\ \max \quad & P(\boldsymbol{x},\lambda) = \sum_{i=1}^{m}\sum_{j=1}^{n}(b_{ij} - (b_{ij}-a_{ij})\lambda)x_{ij} \\ \text{s.t.} \quad & \sum_{j=1}^{n} x_{ij} = S_i \qquad i = 1,\dots,m \\ & \sum_{i=1}^{n} x_{ij} = D_j \qquad j = 1,\dots,n \\ & x_{ij} \geqslant 0 \qquad i = 1,\dots,m;\ j = 1,\dots,n. \end{aligned} \tag{8}$$

Functions $f(\lambda,\boldsymbol{x})$ and $g(\lambda,\boldsymbol{x})$ in the upper-level problem (7) are defined as follows:

$$f(\lambda,\boldsymbol{x}) = \lambda - \left(\beta - \sum_{i=1}^{m}\sum_{j=1}^{n}(\alpha_{ij} + (\beta_{ij}-\alpha_{ij})\lambda)x_{ij}\right)\Big/(\beta-\alpha), \tag{9}$$

$$g(\lambda,\boldsymbol{x}) = \lambda - \left(\sum_{i=1}^{m}\sum_{j=1}^{n}(b_{ij} - (b_{ij}-a_{ij})\lambda)x_{ij} - a\right)\Big/(b-a). \tag{10}$$

It's obvious that the lower-level problem (8) can be considered as a bi-objective parametric programming problem, with λ as a parameter. Keshavarz and Khorram (2011) attempted to find the solution of the bi-level programming problem (7) by finding and comparing the optimal solutions of two distinct bi-level programming problems, which upper-level problems of them are same as (7), but the lower-level's objective of the first one is $\min T(\boldsymbol{x},\lambda)$, and for the latter is $\max P(\boldsymbol{x},\lambda)$. They used a parametric programming approach to solve these problems and finally designed a comparative analysis to find the solution of (7). Their proposed comparative approach tests boundary values of some intervals that maybe contain the optimal λ, and paid no attention to the interior values of intervals. To address this shortcoming, in the next section, a revised algorithm is designed and numerically improved the solution of their illustrative example.

3. A revised algorithm

Keshavarz and Khorram considered the following bi-level programming problems (Models (22) and (23) in Keshavarz & Khorram, 2011).

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & f(\lambda,\boldsymbol{x}) \leqslant 0, g(\lambda,\boldsymbol{x}) \leqslant 0, \quad f(\lambda,\boldsymbol{x}) \cdot g(\lambda,\boldsymbol{x}) = 0, \\ & \boldsymbol{x} \in \operatorname{argmin}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}(\alpha_{ij} + (\beta_{ij}-\alpha_{ij})\lambda)x_{ij} : \boldsymbol{x} \in X\right\} \end{aligned} \tag{11}$$

$$\begin{aligned} \max \quad & \lambda \\ \text{s.t.} \quad & f(\lambda,\boldsymbol{x}) \leqslant 0, g(\lambda,\boldsymbol{x}) \leqslant 0, f(\lambda,\boldsymbol{x}) \cdot g(\lambda,\boldsymbol{x}) = 0, \\ & \boldsymbol{x} \in \operatorname{argmax}\left\{\sum_{i=1}^{m}\sum_{j=1}^{n}(b_{ij} - (b_{ij}-a_{ij})\lambda)x_{ij} : \boldsymbol{x} \in X\right\} \end{aligned} \tag{12}$$

Let $(\lambda^*,\boldsymbol{x}^*)$, $(\lambda_f^*,\boldsymbol{x}_f^*)$ and $(\lambda_g^*,\boldsymbol{x}_g^*)$ be the optimal solutions of the models (7), (11) and (12), respectively. It is obvious that $\boldsymbol{x}_f^*$ and $\boldsymbol{x}_g^*$ are efficient solutions of the model (8), therefore $(\lambda_f^*,\boldsymbol{x}_f^*)$ and $(\lambda_g^*,\boldsymbol{x}_g^*)$ are feasible solutions of the model (7), and so $\lambda^* \geqslant \max\{\lambda_f^*,\lambda_g^*\}$.

Keshavarz and Khorram's proposed algorithm finds $(\lambda_f^*,\boldsymbol{x}_f^*)$ and $(\lambda_g^*,\boldsymbol{x}_g^*)$, by a parametric programming approach; final step of this algorithm suggests the value $\max\{\lambda_f^*,\lambda_g^*\}$ as the optimal value of (7), but this is not true generally, in fact $\max\{\lambda_f^*,\lambda_g^*\}$ is a lower bound for λ^*. In order to overcome this shortcoming, we formulate the following problem.

$$\Pr(\lambda): \begin{cases} \max \quad s_1 + s_2 & \text{(a)} \\ \text{s.t.} \quad \lambda + s_1 = \frac{\beta - \sum_{i=1}^{m}\sum_{j=1}^{n}(\alpha_{ij}+(\beta_{ij}-\alpha_{ij})\lambda)x_{ij}}{(\beta-\alpha)} & \text{(b)} \\ \qquad \lambda + s_2 = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}(b_{ij}-(b_{ij}-a_{ij})\lambda)x_{ij} - a}{(b-a)} & \text{(c)} \\ \qquad \sum_{j=1}^{n} x_{ij} = S_i, \quad i = 1,\dots,m & \text{(d)} \\ \qquad \sum_{i=1}^{n} x_{ij} = D_j, \quad j = 1,\dots,n & \text{(e)} \\ \qquad x_{ij} \geqslant 0, \quad i = 1,\dots,m; j = 1,\dots,n & \text{(f)} \\ \qquad s_1, s_2 \geqslant 0 & \text{(g)} \\ \qquad 0 \leqslant \lambda \leqslant 1 & \text{(h)} \end{cases} \tag{13}$$

Constraints (13.b) and (13.c) are manipulated versions of $f(\lambda,\boldsymbol{x}) \leqslant 0$ and $g(\lambda,\boldsymbol{x}) \leqslant 0$, respectively. Referring to (9) and (10), we see that s_1 and s_2 are slack variables associated with the constraints. It should be noted that the problem (13) is a non-linear programming problem with λ, s_1, s_2 and $\boldsymbol{x} = (\dots, x_{ij}, \dots)$ as decision variables, but for a fixed value of λ this problem is a linear programming problem. Furthermore, if $\bar{\boldsymbol{x}} = (\dots, \bar{x}_{ij}, \dots)$ is an arbitrary feasible solution of the model (1) then $(\lambda, s_1, s_2, \boldsymbol{x}) = \left(0, \frac{\beta - \sum_{i=1}^{m}\sum_{j=1}^{n}\alpha_{ij}\bar{x}_{ij}}{(\beta-\alpha)}, \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}b_{ij}\bar{x}_{ij} - a}{(b-a)}, \bar{\boldsymbol{x}}\right)$ is a feasible solution of the model (13), and so this model is always feasible.

Following theorems show two important properties of the model (13).

Theorem 1. *Let $\lambda \in [0,1]$ be a fixed value, if $(s_1^\lambda, s_2^\lambda, \boldsymbol{x}^\lambda)$ is an optimal solution of the model (13), then $\boldsymbol{x}^\lambda$ is an efficient solution for (8).*