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Change-point detection for shifts in control charts using fuzzy shift change-point algorithms ☆

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ABSTRACT

Knowing the real time of changes, called change-point, in a process is essential for quickly identifying and removing special causes. Many change-point methods in statistical process control assume the distribution and the in-control parameters of the process known, however, they are rarely known accurately. Small errors accompanied with estimated parameters may lead to unfavorable change-point estimates. In this paper, a new method, called fuzzy shift change-point algorithm, which does not require the knowledge of the distribution nor the parameter of the process, is proposed to detect change-points for shifts in process mean. The fuzzy c-partition concept is embedded into change-point formulation in which any possible collection of change-points is considered as a partitioning of data with a fuzzy membership. These memberships are then transferred into the pseudo memberships of observations belonging to each individual cluster, so the fuzzy c-means clustering can be used to obtain the estimates for shifts. Subsequently, the fuzzy c-means algorithm is used again to obtain new iterates of change-point collection memberships by minimizing an objective function concerning the deviations between observations and the corresponding cluster means. The proposed algorithm is nonparametric and applicable to normal and non-normal processes in both phase I and II. The performance of the proposed fuzzy shift change-point algorithm is discussed in comparison with powerful statistical methods through extensive simulation studies. The results demonstrate the superiority and usefulness of our proposed method.

1. Introduction

Control charts are most widely used for process monitoring. However, due to the potential delay in producing signals, control charts do not disclose the real time of process changes, which is essential for engineers to identify the special causes and in turn hasten the implementation of appropriate corrective action. Hence, detection for the time of changes, called change-point (CP), in a process is important in quality control. Process monitoring usually involves two phases of analyses. In a preliminary analysis called phase I analysis, data are collected and analyzed retrospectively; the work in phase I involves detecting the presence of CPs and estimating the in-control process parameters. In phase II, the in-control process parameters are assumed accurately estimated or known. Hence, much of the work in CP detection has focused on "known parameters" problems, however, the true values are rarely known in practice.

There have been many researches in developing CP methods for different quality characteristics. The likelihood approach has been adopted most often in CP estimation such as Eyvazian, Noorossana, Saghaei, and Amiri (2011), Shams, Ajorlou, and Yang (2013), Niaki and Khedmati (2013) and Niaki and Khedmati (2014), Bae, Mun, and Kim (2013), and Dogu (2014). Moreover, a CP framework for detecting distributional changes in a normal process sequentially were first introduced by Hawkins, Qiu, and Kang (2003) and it had been extended in various ways by Ross, Adams, Tasoulis, and Hand (2011) and Ross and Adams (2012). These CP detection methods are likelihood based approaches which require knowing the distribution of the process. Furthermore, a likelihood estimator is only applicable to the processes following the distribution which is based on, for example, a likelihood estimator for normal processes is not applicable for Poisson processes. Therefore, nonparametric methods for CP detection were considered by some researchers such as Zhou, Zou, Zhang, and Wang (2009), Huang, Kong, and Huang (2014), and Sharkey and Killick (2014). More

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recently, artificial neural network was applied to CP detection, for instance, Ghiasabadi, Noorossana, and Saghaei (2013), Movaffagh and Amiri (2013), Amiri, Niaki, and Moghadam (2014).

On the other hand, locating CPs in a process is similar to partitioning data into clusters of similar individuals, hence, clustering techniques have also been employed to detect CPs in some studies such as Ghazanfari, Alaeddini, Niaki, and Aryanezhad (2008), Harnish, Nelson, and Runger (2009), and Shams et al. (2013). Most clustering researches utilized hard clustering methods in statistics which restrict each data point belonging to exactly one cluster. However, there are usually less crisp definite boundaries between clusters in real datasets so that fuzzy partitioning which allows data points belonging to more than one cluster is generally better suited in real applications. Fuzzy logic is a natural way to deal with the vagueness of data (Zadeh, 2008). In particular, fuzzy clustering has been widely studied and applied in a variety of substantive areas (see Bezdek, 1981; Hoppner, Klawonn, Kruse, & Runkler, 1999; Yang, 1993). Fuzzy control charts based on fuzzy set theory are well-documented in literature such as Faraz and Shapiro (2010), Wang, Li, and Yasuda (2014), and Sentürk, Erginel, Kaya, and Kahraman (2014), however, as we know, there was less research in use of fuzzy clustering methods for CP detection. Although Alaeddini, Ghazanfari, and Aminnayeri (2009) mentioned using a revised fuzzy *c*-means and a revised entropy clustering method in detecting a step change, however, the performances of two methods in their simulation are not good, even though they did not explain how the modifications were made. Alaeddini et al. (2009), Zarandi and Alaeddini (2010), Kazemi, Bazargan, and Yaghoobi (2014) used a hybrid fuzzy-statistical clustering method to detect a CP of normal processes in phase II. In carrying out the hybrid fuzzy-statistical clustering algorithm, in-control process parameters were assumed known, but they are rarely known accurately. Although the usage of their approach can be extended to phase I problems, its performance was affected seriously by the variability of the estimates of parameters and it cannot be used for detecting multiple CPs.

In this paper, we embed fuzzy clustering with fuzzy *c*-partition into a CP framework to detect the time of shifts in mean, and simultaneously produce the estimates of shifts in different segments in which multiple CPs are allowed. The distribution of the process is not needed to know. Furthermore, the proposed method does not require the true values of in-control process parameters nor their estimates obtained in a phase I study, and it is applicable to both phases. Besides, multiple shifts may occur in a phase I process, but the control charts fail to detect the presence of any shifts (see Sullivan, 2002). Although a method for detecting a single CP may be employed to detect multiple CPs through binary segmentation, Hawkins (2001) pointed out "the hierarchic binary splitting, though fast, usually fails to give the optimum splits if there are two or more of them". Instead of detecting multiple CPs one at a time, the proposed method can be used for detecting multiple CPs simultaneously in a process.

The remainder of this paper is organized as follows. The fuzzy *c*-means clustering algorithm is reviewed in Section 2. Then, the fuzzy shift change-point algorithm is proposed in Section 3. The effectiveness of the proposed algorithm is examined through extensive simulation with numerical data and real datasets in Section 4. Finally, conclusions are stated in Section 5.

2. Fuzzy *c*-means clustering and $\overline{X}$ charts

Let $\mathbf{X} = \{\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n\}$ be a given data set in a *p*-dimensional real space $\Re^p$. We would like to partition the dataset $\mathbf{X}$ into *c* subsets that can well represent the data structure of $\mathbf{X}$. The partition of *c*clusters can be described by a $c \times n$ partition matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_c]^T = [\mu_{ij}]_{c\times n}$ where each element μ_{ij} of $\mathbf{U}$ represents the membership of $\mathbf{x}_j$ belonging to the *i*th cluster. In general, there are three kinds of partition matrices used in clustering: (1) The hard *c*-partitions U_H with $\mu_{ij} \in \{0, 1\}$ for all *i* and *j* and $\sum_{i=1}^{c}\mu_{ij} = 1$ for each *j*; (2) The fuzzy *c*-partitions U_F with $\mu_{ij} \in [0, 1]$ for all *i* and *j* and $\sum_{i=1}^{c}\mu_{ij} = 1$ for each *j*; and (3) The possibilistic *c*-memberships U_P with $\mu_{ij} \in [0, 1]$ for all *i* and *j* and $\sum_{i=1}^{c}\mu_{ij} > 0$ for each *j*. The best-known clustering algorithm with hard *c*-partitions U_H is k-means (or called hard *c*-means). The fuzzy *c*-means (FCM) clustering algorithm with fuzzy *c*-partitions U_F is a well-known fuzzy extension of k-means.

The FCM is an iterative algorithm by using the necessary conditions for minimizing the following objective function, J_{FCM}:

$$J_{FCM}(\mathbf{U}, \mathbf{A}) = \sum_{i=1}^{c}\sum_{j=1}^{n}\mu_{ij}^m d_{ij}^2, \tag{1}$$

where the weighting exponent $m > 1$ is a fuzziness index; $\mu_{ij} \in U_F$ are fuzzy *c*-partitions; $\mathbf{A} = \{\mathbf{a}_1, \ldots, \mathbf{a}_c\}$ over the real space $\Re^p$ is the set of *c* cluster centers, and d_{ij} is a dissimilarity measure where $d_{ij} = \sqrt{d^2(\mathbf{x}_j, \mathbf{a}_i)} = \|\mathbf{x}_j - \mathbf{a}_i\|$ is the Euclidean distance between $\mathbf{x}_j$ and $\mathbf{a}_i$. Note that other types of dissimilarity $d(\mathbf{x}_j, \mathbf{a}_i)$ may be used to improve the usage and effectiveness of FCM. The updated equations for minimizing J_{FCM} are as follows (see Bezdek, 1981; Yang, 1993):

$$\mu_{ij} = \frac{(d_{ij}^2)^{-1/(m-1)}}{\sum_{k=1}^{c}(d_{kj}^2)^{-1/(m-1)}}, \quad i = 1, \ldots, c, \quad j = 1, \ldots, n. \tag{2}$$

$$a_i = \frac{\sum_{j=1}^{n}\mu_{ij}^m x_j}{\sum_{j=1}^{n}\mu_{ij}^m}, \quad i = 1, \ldots, c. \tag{3}$$

3. Fuzzy shift change-point algorithm

Let $\{x_1, x_2, \ldots, x_T\}$ be a sequence of process readings. Assume that $c-1$ abrupt changes occur at unknown time points, $\{\tau_1, \tau_2, \ldots, \tau_{c-1}\}$ with $1 \leqslant \tau_1 < \cdots < \tau_{c-1} \leqslant T-1$. Define $\tau_0 = 0$ and $\tau_c = T$. The work here focuses on detecting the time of shifts in mean only, i.e., $a_i \neq a_j$, for $1 \leqslant i \neq j \leqslant c$ but $\sigma_i^2 = \sigma^2$, for all $1 \leqslant i \leqslant c$. Since changes can occur randomly among the time points, $\{1, \ldots, T-1\}$, we may consider each combination of $c-1$ points drawn from $\{1, \ldots, T-1\}$, as a CP combination with a fuzzy membership, say α. By transferring these memberships into the pseudo memberships of data points belonging to each cluster, respectively, we then use the FCM clustering twice to obtain the estimates for the means of the *c* clusters and the new iterates of CP memberships respectively. Finally, the optimal CPs and the estimates for the means of *c* segments can be derived by repeating these two procedures. In the following, we first consider one-CP models and subsequently multiple-CP models.

3.1. Fuzzy shift change-point algorithm for one-change-point models

Assume that the successive process readings $\{x_1, x_2, \ldots, x_\tau, \ldots, x_T\}$ follows a distribution (for instance, normal) till time τ, after τ, the mean of the distribution shifts but the variance remains unchanged. Obviously, $\tau \in \{1, \ldots, T-1\}$. From the viewpoint of probability, suppose that $P(\tau = i) = \alpha_i, \tau \in \{1, \ldots, T-1\}$ with $\sum_{i=1}^{T-1}\alpha_i = 1, 0 \leqslant \alpha_i \leqslant 1$, then the probability of any point x_j, $j = 1, \ldots, T-1$, belonging to the first group can be calculated as follows: