Computers & Industrial Engineering 93 (2016) 55–66

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Robust parameter optimization based on multivariate normal boundary intersection

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ARTICLE INFO

Article history:
Received 11 April 2015
Received in revised form 22 December 2015
Accepted 24 December 2015
Available online 4 January 2016

Keywords:
Global optimization
Multivariate statistics
Normal Boundary Intersection (NBI)
Multi-objective Robust Parameter Design (MRPD)
Principal Component Analysis (PCA)

ABSTRACT

Normal Boundary Intersection (NBI) is traditionally used to generate equally spaced and uniformly spread Pareto Frontiers for multi-objective optimization programming (MOP). This method tends to fail, however, when correlated objective functions must be optimized using Robust Parameter Designs (RPD). In such multi-objective optimization programming, there can be reached impractical optima and non-convex frontiers. To reverse this shortcoming, it is common to apply Principal Component Analysis (PCA), which provides uncorrelated objective functions. The aim of this paper is to combine the Robust Parameter Designs, Principal Component Analysis, and Normal Boundary Intersection approaches into a novel method called RPD-MNBI. This approach finds equally spaced Pareto optimal frontiers that are capable of minimizing noise variables' effects. To validate this proposal, this study investigates an end milling process. The most important empirical finding is that the original correlation structure is preserved. On the other hand, the Weighted Sums and Normal Boundary Intersection-Mean Square Error methods, modify the process behavior, resulting in unreal optima. Finally, confirmation runs using an L9 Taguchi design were performed for 10%, 50%, and 90% weights. The proposed method provides process robustness according to confidence intervals for both mean and standard deviation.

1. Introduction

Normal Boundary Intersection (NBI) (Das & Dennis, 1998) was developed mainly to compensate for the shortcomings attributed to the method of weighted sums (WS) in multi-objective optimization programming (MOP). NBI addresses the WS method's inability to find—even when using a uniform spread-of-weight vector—a uniform spread of Pareto optimal solutions. It is possible when using such a vector for there to emerge a non-convex Pareto set with missed Pareto points on the concave parts of the trade-off surface. A pivotal aspect in MOP problems is the presence of strong correlations among the several estimated response surfaces. This aspect of multivariate optimization is very common in the industry and generally promotes unstable regression models and standard errors of coefficients. As a result, the obtained optimization results could be unreal (Bratchell, 1989; Brito, Paiva, Ferreira, Gomes, & Balestrassi, 2014; Govindaluri & Cho, 2007; Jeong, Kim, & Chang, 2005; Kovach & Cho, 2009; Lee & Park, 2006; Paiva et al., 2012; Paiva, Paiva, Ferreira, Balestrassi, & Costa, 2009; Shaibu & Cho, 2009; Shin, Samanlioglu, Cho, & Wiecek, 2011; Tang & Xu, 2002; Wu, 2005; Yuan, Wang, Yu, & Fang, 2008).

To find optimal Pareto solutions, Ahmadi, Moghimi, Esmaeel, Agelidis, and Sharaf (2015) addressed a multi-objective electric model to integrate the generation of thermal units considering heat and power dispatch. To achieve these goals, the researchers employed the NBI method to find the optimal Pareto solution as the best tradeoff between cost, green-house gas emission and heat generation. Aalae, Abderrahmane, Gael, and Olivier (2015) performed a coupling between the NBI algorithm with Radial Basis Function (RBF) to create a simple tool with a reasonable calculation time to solve multi-criteria optimization problems. Their approach was able to efficiently solve the multi-criteria shape optimization problem of structures with nonlinear behavior. Largo, Zhang, and Vega-Rodríguez (2014) applied NBI with a version of the Multi-objective Evolutionary Algorithm based on Decomposition (MOEA/D) for solving tri-objective optimization problems of telecommunication with objectives. The authors argued that their

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http://dx.doi.org/10.1016/j.cie.2015.12.023
0360-8352/

List of acronyms

DOE	design of experiment
LCB	lower confidence bound
MNBI	multivariate normal boundary intersection
MOEA/D	Multi-objective Evolutionary Algorithm based on Decomposition
MSE	mean square error
OLS	ordinary least squares
QC	Quality Characteristics
RPD	Robust Parameter Designs
UCB	upper confidence bound
WMMSE	Weighted Multivariate Mean Square Error
DRS	dual response surface
MMSE	multivariate mean square error
MOP	multi-objective optimization programming
MRPD	Multi-objective Robust Parameter Design
NBI	Normal Boundary Intersection
PCA	Principal Component Analysis
RBF	Radial Basis Function
RSM	response surface methodology
WLS	Weighted Least Squares
WS	weighted sums

procedure had shown very promising results in real-world telecommunication problems with multiple objective functions.

In addition to the aforementioned papers, Izadbakhsh, Gandomkar, Rezvani, and Ahmadi (2015) employed the NBI approach to simultaneously minimize the cost and emission for an economic/environmental model for optimal energy management. Oujebbour, Habbal, Ellaia, and Zhao (2014) applied NBI and Normalized Normal Constraint Method to generate a set of Pareto-optimal solution in a MOP problem of a stamping process. Ganesan, Vasant, and Elamvazuthi (2013) used the NBI method to generate optimal solutions to the green sand process. Optimizing multiple responses of rotor-bearing systems, Lopez, Ritto, Sampaio, and de Cursi (2014) came up with a new robust optimization algorithm based on NBI and the penalization of mean and variance for dealing with non-convex MOP.

Given the findings of these papers, it can be seen that the correlation influence becomes more important in the context of Pareto Frontiers. Such an effect should be taken into consideration. Since the procedure weighs two or more objective functions, if the correlation is strong and neglected, the procedure's weights will promote an impractical separation in responses. The natural correlation structure can, as a result of such separation, be highly affected. One could obtain, in other words, an excellent Pareto Frontier composed of unrealistic feasible solutions. To avoid such a Frontier inconsistency, uncorrelated objective functions should be obtained through response surface designs and principal component analyses (PCA). This multivariate technique produces a new dataset, one that extracts eigenvalues and eigenvectors from either a covariance or correlation matrix. It is with the new dataset that uncorrelated response surface models may be built.

Another essential issue in industrial problems is the simultaneous optimization of mean and variance. The combined array is an efficient design of experiment (DOE) approach where the noise variables are inserted into the matrix of control variables, generally represented by a central composite design (Montgomery, 2009). Once the dependent variable (Y) is measured, a full quadratic model, $Y = f(\mathbf{x}, \mathbf{z})$, is estimated by using the ordinary least squares (OLS) algorithm. After that, mean and variance equations can be obtained by taking partial derivatives of the estimated response surface ($\widehat{Y}$) with respect to the noise factors ($\mathbf{z}$).

The noise variables' effect can be expressed as a variance equation. This optimization approach is called Multi-objective Robust Parameter Design (MRPD). The simplest MRPD problem is a bi-objective optimization problem where the two objective functions are the mean and the variance. Suppose now there are ξ estimated models for the mean and the same number of models for the variance. The MOP problem will then have 2ξ estimated equations and ξ dual response surfaces. It thus becomes a non-trivial task to carry out the industrial compromise of offering quality products by employing the simultaneous optimization of blocks of mean and variance equations (Kazemzadeh, Bashiri, Atkinson, & Noorossana, 2008).

This study differs from the most commonly proposed RPD approaches in that the weights of the mean and variance equations of the principal component scores can be obtained from a control-noise response surface equation. Moreover, the weighted approach of principal components has already been used successfully by several authors (Gomes, Paiva, Costa, Balestrassi, & Paiva, 2013; Lopes et al., 2013; Peruchi, Balestrassi, Paiva, Ferreira, & Carmelossi, 2013). Besides reducing dimensions, the RPD-MNBI (robust parameter optimization based on multivariate normal boundary intersection) method has two advantages: (i) It considers the correlation among the multiple responses and (ii) it generates convex Pareto Frontiers of the mean (f_μ) and variance ($f_{\sigma 2}$) functions. Accordingly, this paper presents the RPD-MNBI method, a multi-objective hybrid approach of RPD that couple NBI with PCA for combined arrays while considering the correlation structure among the response variables. To illustrate the proposal, this study uses a case study of a bivariate AISI 1045 steel end milling operation. The optimization results are statistically validated, confirming the adequacy of the paper's proposal.

2. Multi-objective optimization and the NBI technique

Normal Boundary Intersection method (NBI), a MOP procedure developed by Das and Dennis (1998), was intended to compensate for the shortcomings attributed to the WS method. According to Shukla and Deb (2007), the WS approach was unable to come up with a uniform spread of Pareto optimal solutions, even if a uniform spread of weighted vectors had been used. Vahidinasab and Jadid (2010) also found that if the Pareto set was non-convex, the Pareto points on the concave parts of the trade-off surface would be missed. This led to Ganesan et al. (2013) finding that when solving non-convex MOP, the NBI approach was deemed to be a more interesting alternative to the WS method.

To use the NBI method, it is necessary to find the payoff matrix Φ by a calculation based on the individual minimum of each objective function. The solution that minimizes the ith objective function $f_i(x)$ will be denoted by $f_i^*(x_i^*)$. $f_i(x_i^*)$ which is obtained when the individual optimal solution x_i^* is substituted in the objective functions. The payoff matrix Φ is shown in Eq. 1.

$$\Phi = \begin{bmatrix} f_1^*(x_1^*) & \cdots & f_1(x_i^*) & \cdots & f_1(x_m^*) \\ \vdots & \ddots & & & \vdots \\ f_i(x_1^*) & \cdots & f_i^*(x_i^*) & \cdots & f_i^*(x_m^*) \\ \vdots & & & \ddots & \vdots \\ f_m(x_1^*) & \cdots & f_m(x_i^*) & \cdots & f_m^*(x_m^*) \end{bmatrix} \Rightarrow \bar{\Phi} = \begin{bmatrix} \bar{f}_1 & \cdots & \bar{f}_1 & \cdots & \bar{f}_1(x_m^*) \\ \vdots & \ddots & & & \vdots \\ \bar{f}_i & \cdots & \bar{f}_i & \cdots & \bar{f}_i(x_m^*) \\ \vdots & & & \ddots & \vdots \\ \bar{f}_m(x_1^*) & \cdots & \bar{f}_m(x_i^*) & \cdots & \bar{f}_m(x_m^*) \end{bmatrix} \quad (1)$$