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Group decision making with incomplete intuitionistic preference relations based on quadratic programming models

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ABSTRACT

This paper presents a quadratic-program-based framework for group decision making with incomplete intuitionistic preference relations (IPRs). The framework starts with introducing a notion of additive consistency for incomplete IPRs, followed by a two-stage quadratic program model for estimating missing values in an incomplete IPR. The first stage aims to minimize inconsistency of the completed IPR and control hesitation margins of the estimated judgments within an acceptable threshold. The second stage is to find the most suitable estimates without changing the inconsistency level. Subsequently, a parameterized formula is proposed to transform normalized interval fuzzy weights into additively consistent IPRs. Two quadratic programs are developed to generate interval fuzzy weights from a complete IPR. The first model obtains interval fuzzy weight vectors by minimizing the squared deviation between the two sides of the transformation formula. By optimizing the parameter value, the second model finds the best weight vector based on the optimal solutions of the first model. A procedure is then developed to solve group decision problems with incomplete IPRs. A numerical example and a group selection problem for enterprise resource planning software products are provided to demonstrate the proposed models.

1. Introduction

In multi-criteria decision making (MCDM), decision-makers (DMs) often employ pairwise comparison to elicit their preference over alternatives. These preference judgments are structured as multiplicative preference relations in the classic analytic hierarchy process (AHP) (Saaty, 1980). To express DMs' pairwise judgments with vagueness, Orlovski (1978) introduced fuzzy preference relations, which is also referred to as reciprocal preference relations (De Baets & De Meyer, 2005; Chiclana, Herrera-Viedma, Alonso, & Herrera, 2009). Crisp-ratio and unit-interval bipolar scales are two most commonly used approaches in representing a DM's pairwise comparison results. The classical AHP adopts a crisp-ratio approach where the numerical value 1 plays a neutral role in representing the DM's indifference between two alternatives. On the other hand, a unit-interval bipolar scale uses the numerical value 0.5 to express its neutral value. This scale has been widely applied to decision models with [0, 1]-valued reciprocal preference relations and [0, 1]-valued interval reciprocal preference relations. It is noted that there exists an isomorphism between a unit-interval bipolar scale with the neutral value 0.5 and a crisp-ratio bipolar scale with the neutral value 1.

A variety of methods have been put forward to generate priority weights from fuzzy preference relations and estimate missing values for incomplete fuzzy preference relations. For instance, Xu (2004) introduced additive consistency and multiplicative consistency for incomplete fuzzy preference relations and developed two goal programs for obtaining priority weights from incomplete fuzzy preference relations. Herrera-Viedma, Chiclana, Herrera, and Alonso (2007) introduced an additive consistency index to define the inconsistency level of a fuzzy preference relation, and put forward an iterative procedure to estimate unknown values for incomplete fuzzy preference relations. Liu, Pan, Xu, and Yu (2012) developed a least square model to determine missing values for incomplete fuzzy preference relations based on additive transitivity.

An element in a fuzzy preference relation represents a DM's judgment with a membership degree. Sometimes, DMs may have hesitancy or uncertainty for their membership judgments. In this situation, Atanassov's (1986) intuitionistic fuzzy sets (A-IFSs) appears to be a convenient representation framework. A-IFSs

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employ both membership and nonmembership functions to characterize DMs' vague judgments, and have been widely applied to areas such as decision making (Qi, Liang, & Zhang, 2015; İntepe, Bozdag, & Koc, 2013; Xu & Liao, 2014), clustering analysis (Chaira, 2011) and machine learning (Szmidt, Kacprzyk, & Bujnowski, 2014). Since Xu (2007) introduced the notion of intuitionistic preference relations (IPRs), decision modeling with IPRs has attracted attention from many researchers in recent years (Jiang, Xu, & Gao, 2015; Xu & Liao, 2014; Yue & Jia, 2015).

Based on various transitivity conditions, some approaches have been devised to estimate missing values in incomplete IPRs and obtain priority weights from complete IPRs. For instance, Xu, Cai, and Szmidt (2011) introduced a multiplicative transitivity equation to define consistency of IPRs and proposed two algorithms to determine missing elements for incomplete IPRs. Gong, Li, Zhou, and Yao (2009) established goal programming models for deriving interval priority weights from IPRs. Xu (2012) put forward an approach to determine interval weights of IPRs based on an error analysis idea. Recently, Xu and Liao (2014) extended crisp and fuzzy AHPs to the intuitionistic AHP and developed a normalizing rank summation method to obtain priority weights from IPRs. Wu and Chiclana (2014) proposed a different multiplicative consistency definition for IPRs and furnished a consistency based procedure to estimate missing values. Wang (2015) revealed that the multiplicative consistency given by Xu et al. (2011) has an undesirable property: the same IPR's consistency status may change when the alternatives are re-labeled. A geometric consistency definition was proposed for IPRs to address this issue. A logarithmic-least-square optimization model was also developed to elicit interval fuzzy weights from IPRs.

Chiclana et al. (2009) converted Tanino's (1984) multiplicative transitivity constraint to an equivalent Cross Ratio uninorm based functional equation for fuzzy preference relations, and indicated that the uninorm-based function is more appropriate to tackle the boundary problem for consistency of reciprocal preference relations. However, as an alternative notion, additive consistency remains a viable choice to characterize whether pairwise comparison judgments are consistent and was adopted in recent research (Cabrerizo, Heradio, Pérez, & Herrera-Viedma, 2010; Meng & Chen, 2015; Zhang, Ma, Li, Liu, & Liu, 2014). As Xu, Li, and Wang (2014) pointed out, the uninorm-based function does not perform well and may yield counterintuitive consistent judgment when a furnished preference value approaches 0 or 1. On the other hand, additive transitivity behaves well with intuitionistic judgments close to $(1,0)$ and $(0,1)$. The authors contemplate that additive and multiplicative consistency might reflect different human cognitive characteristics when they provide their pairwise judgments: For linear-thinking-inclined DMs, additive consistency is more appropriate, but for nonlinear thinking DMs, multiplicative consistency appears to be a better choice. The research herein adopts the notion of additive consistency.

Under the framework of additive consistency, Xu (2009) introduced a feasible region method to define additively consistent IPRs and established a linear program to obtain a priority weight vector from an IPR. Gong, Li, Forrest, and Zhao (2011) presented a goal program and a least square model for deriving interval fuzzy weights from IPRs. Wang (2013) introduced a new transitivity condition to define additively consistent IPRs and developed two goal programs for deriving intuitionistic fuzzy priority weight vectors from IPRs. In Gong et al. (2011) and Wang (2013), the coefficient of the transformation formulae between additively consistent IPRs and priority weights is assumed to be 0.5, same as that of Tanino's (1984) additive transformation formula. It has been found that this transformation relation is not always valid (Fedrizzi & Brunelli, 2009; Liu et al., 2012; Xu, Da, & Liu, 2009; Xu, Da, & Wang, 2010; Xu et al., 2014; Hu, Ren, Lan, Wang, & Zheng, 2014). This motivates us to introduce a parameterized transformation formula between additively consistent IPRs and priority weights and develop a corresponding priority weight derivation method.

This research first extends the additive consistency for IPRs to the case of incomplete IPRs. A two-stage quadratic program framework is then put forward to estimate missing values in incomplete IPRs. The first stage minimizes the inconsistency level of the completed IPR with an appropriate control of hesitation margins of the estimated judgments. The second stage finds the most suitable estimated values among the results obtained from the first stage without changing the inconsistency level. By analyzing the inherent relationship between an additively consistent IPR (Wang, 2013) and a normalized interval fuzzy weight vector and introducing a parameterized transformation formula, two quadratic programs are developed to obtain a normalized interval fuzzy weight vector. The first model minimizes the squared deviations between the original intuitionistic judgments and the parameterized interval-weight-based preference values. The second model identifies the most appropriate interval fuzzy weight vector among the optimal solutions in the first model by optimizing the parameter value. Finally, by applying the aforesaid models, a procedure is developed for solving group decision making (GDM) problems with incomplete IPRs.

The remainder of the paper is organized as follows. Section 2 reviews basic concepts of additively consistent fuzzy preference relations and IPRs. Section 3 introduces the notion of additive consistency for incomplete IPRs, and devises a two-stage approach to estimate missing values in incomplete IPRs. Two quadratic programs are proposed for generating interval fuzzy weights from complete IPRs in Section 4. Section 5 puts forward a practical procedure to solve GDM problems with incomplete IPRs, followed by a numerical illustration. Conclusions are drawn in Section 6.

2. Preliminaries

This section presents basic concepts of additively consistent fuzzy preference relations and IPRs.

Let $X = \{x_1, x_2, \ldots, x_n\}$ be a collection of n alternatives. A fuzzy preference relation (Orlovski, 1978) on X is defined by a pairwise judgment matrix $R = (r_{ij})_{n\times n}$, where r_{ij} indicates a DM's fuzzy preference of alternative x_i over x_j such that

$$r_{ij} \in [0,1],\ r_{ij} + r_{ji} = 1,\ r_{ii} = 0.5, \quad \forall i,\ j = 1,2,\ldots,n \tag{2.1}$$

Definition 2.1 Tanino, 1984. A fuzzy preference relation $R = (r_{ij})_{n\times n}$ is additively consistent if R satisfies additive transitivity:

$$r_{ij} = r_{ik} - r_{kj} + 0.5, \quad \forall i,\ j,\ k = 1,2,\ldots,n. \tag{2.2}$$

Due to additive reciprocity $r_{ij} + r_{ji} = 1$, (2.2) is equivalent to

$$r_{ij} + r_{jk} + r_{ki} = r_{ik} + r_{kj} + r_{ji}, \quad \forall i,\ j,\ k = 1,2,\ldots,n. \tag{2.3}$$

Liu et al. (2012) established that $R = (r_{ij})_{n\times n}$ is additively consistent if and only if there exists a normalized priority weight vector $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T,\ \omega_i \geqslant 0,\ i = 1,2,\ldots,n$ and $\sum_{i=1}^{n} \omega_i = 1$, such that

$$r_{ij} = c(\omega_i - \omega_j) + 0.5, \quad \forall i,\ j = 1,2,\ldots,n \tag{2.4}$$

where $c = \max\{0.5, \frac{n}{2} - \min_{1\leqslant i\leqslant n}\{\sum_{k=1}^{n} r_{ik}\}\}$.

As $r_{ii} = 0.5$ for all $i = 1,2,\ldots,n$, one has

$$0.5 \leqslant c \leqslant \frac{n-1}{2} \tag{2.5}$$

It should be noted that multiple normalized priority weight vectors and c values under (2.4) may exist for a given additively consistent