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# Economic production quantity in batch manufacturing with imperfect quality, imperfect inspection, and destructive and non-destructive acceptance sampling in a two-tier market

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ABSTRACT

This article develops an economic production quantity (EPQ) model for the case where the production process and inspection are both not perfect. Unlike the models in the literature, the proposed model aims to find the optimal lot size for a manufacturer who produces items in batches; and the batches are subjected to destructive or non-destructive acceptance sampling process before batches can be sent out to the market. The manufacturer can use destructive testing or non-destructive testing that can best assess the primary quality characteristic. Two errors can happen in this stage: Type 1 and Type 2 errors. If a lot is rejected, it goes through a more expensive non-destructive screening stage to segregate items into non-defective, reworkable, and salvage. Items that reach the primary market and found to be defective are returned, with a return cost to the manufacturer; in this case, returned items are either reworked or sold as salvage. The expected net profit function consists of the following components: primary and secondary market sales, sales of salvage items, setup and variable production cost, return cost, rework cost, screening cost, destructive cost, work-in-process, sales items inventory, rework item inventory, and salvage inventory. For both destructive and non-destructive testing situations, the optimal lot size has been found and the optimality criterion has been tested. The article concludes with a numerical example and sensitivity analysis.



## 1. Introduction

The classical economic production quantity model has been extended to consider more realistic cases. Advances in the research on the lot size problem can be found in Andriolo, Battini, Grubbström, Persona, and Sgarbossa (2014), Cárdenas-Barrón, Chung, and Treviño-Garza (2014), and Glock, Grosse, and Ries (2014). One particular focus of research by scientists is the consideration of the imperfect production process in determining the EPQ/EOQ value. The earliest articles in the study of the effect of the imperfect production process on the EPQ/EOQ are of Rosenblatt and Lee (1986) and Porteus (1986). The literature that treats inventory models with imperfect quality can be broadly classified into two categories: predictable or unpredictable identification of defective items. The literature on the predictable identification of the quality of items is rich. The EPQ model of Salameh and Jaber (2000) considers imperfect items, which can be identified by a screening process and accumulated into a single batch to be sold at the end of the production cycle. Any reviewer of this article must refer to the correction outlined in Cárdenas-Barrón (2000). Goyal and Cárdenas-Barrón (2002) take on the model of Salameh and Jaber (2000) and propose a simple approach for determining the lot size. Recently, Chiu, Lin, Wu, and Yang (2011) and Chiu, Lin, and Chang (2012) propose an EPQ model with multiple deliveries and known proportion of defective items. The production process is followed by a screening stage; the identified defective items are either scrap or reworkable items. Cárdenas-Barrón, Treviño-Garza, Taleizadeh, and Pandian (2015) solve the previous models with integer values for the lot size and the number of shipments. Chiu, Liu, Chiu, and Chang (2011) develop a model to determine the optimal EMQ in an environment where random defects occur and the rework is performed at the end of the production cycle before the lot can be shipped out. The EPQ model of Hu, Xu, and Guo (2011) assumes fuzzy defective rate with two cases of screening rates. Roya, Sanab, and Chaudhuric (2011) study the effect of the inspection time on the stockout and the resulting back ordering in the case of imperfect quality. Al-Salamah and Alsawafy (2012) consider an EOQ model for lots having fractions of scrap and reworkable items; and the demand is satisfied from perfect and reworked items; and the scrap items

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**Notation**

| | |
|---|---|
| $Y$ | production rate (units per year) |
| $D_p$ | primary market demand rate (units per year) |
| $z$ | rework rate (units per year) |
| $Q$ | lot size (units) |
| $Q_D$ | lot size under acceptance sampling with destructive testing (units) |
| $Q_{ND}$ | lot size under acceptance sampling with non–destructive testing (units) |
| $T$ | cycle length (year) |
| $n$ | sample size (units) |
| $\eta$ | probability an item is defective |
| $E_1$ | probability of Type 1 error |
| $E_2$ | probability of Type 2 error |
| $s_p$ | unit profit of sales in the primary market ($ per unit) |
| $s_s$ | unit profit of sales in the secondary market ($ per unit) |
| $s_v$ | unit salvaged item profit ($ per unit) |
| $c_p$ | unit production cost ($ per unit) |
| $c_u$ | production setup cost per cycle ($ per cycle) |
| $c_r$ | unit return cost ($ per unit) |
| $c_w$ | unit rework cost ($ per unit) |
| $c_s$ | unit screening cost ($ per unit) |
| $c_d$ | unit destructive testing cost ($ per unit) |
| $\mu$ | fraction of reworkable items |
| $h_1$ | inventory holding cost of the work-in-process ($ per unit per year) |
| $h_2$ | inventory holding cost per period of items to be shipped to the primary market ($ per unit per year) |
| $h_3$ | inventory holding cost per period of reworked items ($ per unit per year) |
| $h_4$ | inventory holding cost per period of salvage items ($ per unit per year) |
| $E(NP_D)$ | expected net profit for acceptance sampling with destructive testing ($) |
| $E(NP_{ND})$ | expected total profit for acceptance sampling with non-destructive testing ($) |
| $X$ | random variable of the number of defective items in the sample |

are sold at the end of the cycle in a single batch with a salvage cost. Hsua and Hsu (2014) determine the optimal production lot size and backorder quantity for different cases of the fraction of defective and time of the rectifying inspection. Tsai and Wu (2012) propose an EPQ model with imperfect quality and learning effect on the production time and cost; imperfect items are reworked. The EPQ models in Krishnamoorthi and Panayappan (2013) and Sivashankari and Panayappan (2014) permit shortages; defective items can be reworked to convert them into good items. Pal, Sanab, and Chaudhuria (2013) investigate an inventory system where the production process can go from an in-control state to an out-of-control state over a time, which is a random variable. In this model, defective items produced during the out-of-control state can be reworked. Jaber, Zanoni, and Zavanella (2014) consider the two cases that imperfect items can be repaired by a contractor or those items are replaced by good items from a supplier. Cárdenas-Barrón, Sarkar, and Treviño-Garza (2013) develop an EMQ model to determine the number of shipments and lot size when the quality is imperfect. An EPQ model for the joint determination of the number of shipments, replenishment lot size, selling price with rework and multi-shipments is proposed by Taleizadeh, Kalantari, and Cárdenas-Barrón (2015). Treviño-Garza, Castillo-Villar, and Cárdenas-Barrón (2015) derive the optimal integer values for the lot size and the number of shipments for inventory models with defective items.

Inventory models with imperfect quality and imperfect inspection have recently gained notable interest in the literature. Yoo, Kim, and Park (2009) and Hsu and Hsu (2013) develop an inventory model in which items are subjected to a screening stage that can misclassify items due to Type 1 and Type 2 errors. Khan, Jaber, and Bonney (2011) assume an economic order quantity model with imperfect quality and imperfect screening in which items are misclassified with fixed rates. Tai (2013) considers imperfect items and good quality items that can deteriorate. The process by which deteriorated items can be identified is also imperfect; deteriorated items, if received by the customer, will incur a cost to the supplier. Chang, Cheng, and Soong (2015) propose an EOQ model with misclassification errors and under the assumption of permissible delays in payment.

In this article, we extend the literature by developing an economic production quantity (EPQ) model for a manufacturer that operates a batch production facility that can produce items with imperfect quality. The manufacturer produces items in lots; and the lots are subjected to acceptance sampling to assess the quality of the items in the lot. In this analysis, we will consider two kinds of tests to evaluate the quality characteristic: destructive testing and non-destructive testing. The manufacturer supplies production items to a two-tier market: primary and secondary. The primary market buys items that are part of the accepted lot. A rejected lot will go through a more expensive non-destructive screening process to separate items into defective and nondefective. Items that are either part of the rejected lot or reworked by fixing the flaws in them are sold in the secondary market at a lower price. Items that have un-repairable flaws are sold as salvage in a single lot at the end of the cycle. Items that are sold at the primary market and found to be defective are returned with a return cost. Returned items and items identified by the screening process as defective can be of two types: reworkable or salvage. The acceptance sampling process is not perfect; hence, two types of error can happen. The first type is called Type 1 error, which occurs when the lot is judged as unacceptable while it is acceptable. Type 2 error occurs when the lot is found acceptable while it is unacceptable. Reworkable items are shipped to the secondary market immediately as they come available. In this model, to prevent shortages, the manufacturer will carry two inventories; one inventory is for finished items produced during the previous cycle and another inventory is for work-in-process produced during the current cycle. Fig. 1 illustrates the flow of items from the production to the markets. Small manufacturers more likely operate a batch production setup, and examples of items produced in batches include sporting goods and bullets. A similar manufacturing need is faced by a local door handle manufacturer. Door handles are produced in batches because there are verities of door handle designs and the demand is not high enough to warrant continuous production.

## 2. Model formulation

### *2.1. EPQ with destructive testing*

We consider a manufacturer whose production process is not perfect. Therefore, produced items are classified as defective and nondefective. Items are produced in batches or lots; as a consequence, the manufacturer will carry a large work-in-process inven-