Computers & Industrial Engineering 93 (2016) 286–301

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Some new generalized aggregation operators for triangular intuitionistic fuzzy numbers and application to multi-attribute group decision making

Shu-Ping Wan [a,*], Feng Wang [a], Li-Lian Lin [a], Jiu-Ying Dong [b,c]

[a] College of Information Technology, Jiangxi University of Finance and Economics, Nanchang 330013, China
[b] School of Statistics, Jiangxi University of Finance and Economics, Nanchang 330013, China
[c] Research Center of Applied Statistics, Jiangxi University of Finance and Economics, Nanchang 330013, China

ARTICLE INFO

Article history:
Received 28 August 2014
Received in revised form 6 October 2015
Accepted 26 December 2015
Available online 12 January 2016

Keywords:
Multi-attribute group decision making
Triangular intuitionistic fuzzy number
Generalized ordered weighted averaging
Generalized hybrid weighted averaging
Multi-objective programming

ABSTRACT

The aim of this paper is to develop some new generalized aggregation operators for triangular intuitionistic fuzzy numbers (TIFNs) and apply to multi-attribute group decision making (MAGDM) problems. First, the weighted possibility attitudinal expected values of TIFNs are defined and a new method is presented to rank TIFNs considering risk attitude of decision maker (DM). The sensitivity analyses on attitudinal character parameter are given. Then, the triangular intuitionistic fuzzy weighted averaging (TIFWA) operator, ordered weighted averaging (TIFOWA) operator, ordered weighted geometric (TIFOWG) operator and hybrid weighted averaging (TIFHWA) operator are defined. We further develop some new generalized aggregation operators for TIFNs, involving the triangular intuitionistic fuzzy generalized ordered weighted averaging (TIFGOWA) operator and generalized hybrid weighted averaging (TIFGHWA) operator. Some desirable properties for these operators are discussed in detail. Utilizing the TIFGHWA and TIFWA operators, we propose a new method for MAGDM with TIFNs and incomplete weight information. In this method, DMs' weights are determined by Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and the weights of attributes are objectively derived through constructing a multi-objective programming model which is transformed into a linear goal program to solve. Finally, the example analysis of an investment selection example verifies the effectiveness and practicability of the proposed method in this paper.

1. Introduction

Multi-attribute decision making (MADM) has been applied in a variety of real-life problems. With the ever-increasingly complexity in real-world decision problems, it is difficult for individual decision maker (DM) to accurately evaluate all the relevant issues. Therefore, to make decision making more reasonable and reliable, multiple DMs together participate in a complex decision problems (Wan, 2013), which results in the appearance of multi-attribute group decision making (MAGDM).

Due to the influence of subjective factors, the DM sometimes relies on intuition and experience to evaluate the complex attributes of alternatives in the decision problems. Thus, it is often that the DM exhibits some hesitation degrees for the assessments. The intuitionistic fuzzy set (IFS) (Atanassov, 1986) is just a suitable tool to characterize such kind of hesitancy information. Since IFS simultaneously contains membership and non-membership degrees, it is more flexible and practical than fuzzy set (Zadeh, 1965) in dealing with ambiguity and uncertainty (Li, 2011; Wan & Dong, 2015). As a special kind of IFSs defined on the real number set, intuitionistic fuzzy numbers (IFNs), such as triangular IFNs (TIFNs) (Li, 2010; Li, Nan, & Zhang, 2010; Nan, Li, & Zhang, 2010; Wan & Dong, 2014; Wan, Wang, & Dong, 2013), trapezoidal IFNs (TrIFNs) (Wan, 2013; Wang & Zhang, 2009; Wu & Cao, 2012) and interval-valued trapezoidal IFNs (IVTrIFNs) (Wu & Liu, 2013) have been employed to the areas of MADM and MAGDM.

Among these researches, the aggregating techniques, such as weighted arithmetic (or geometric) aggregation operators, are usually used to solve MADM and MAGDM problems with IFNs. For example, Wan et al. (2013) defined the triangular intuitionistic fuzzy weighted averaging (TIFWA) operator and proposed a extended VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) method for solving MAGDM with TIFNs. Wang and Zhang (2009) developed some aggregation operators of TrIFNs including intuitionistic trapezoidal fuzzy weighted arithmetic

* Corresponding author at: College of Information Technology, Jiangxi University of Finance and Economics, No. 168, Shuang-gang East Road, Economic and Technological Development District, Nanchang 330013, China. Tel./fax: +86 0791 13870620534.
E-mail address: shupingwan@163.com (S.-P. Wan).

http://dx.doi.org/10.1016/j.cie.2015.12.027
0360-8352/

averaging operator and weighted geometric averaging operator, which are applied to MADM problems. Wu and Cao (2012) defined some geometric aggregation operators of TrIFNs and proposed an approach to the MAGDM with TrIFNs. Wan (2013) defined some power average operators of TrIFNs and applied to MAGDM. Wan and Dong (2015) defined power geometric operators of TrIFNs and developed four kinds of methods for MAGDM. Wu and Liu (2013) investigated the attitudinal score and accuracy expected functions for IVTrIFNs and defined some geometric operators for IVTrIFNs. Based on these expected functions and aggregating operators for IVTrIFNs, they proposed a method for MAGDM with IVTrIFNs.

In addition, as an important issue on the investigation of IFNs, the ranking method of IFNs has attracted considerable attention of some researchers. For example, Nan et al. (2010) utilized the average indexes of membership and non-membership functions to rank TIFNs and then studied matrix game with payoffs of TIFNs. Li (2010) defined the value and ambiguity for a TIFN, developed a new ranking method based on the ratio of the value index to the ambiguity index and then applied to MADM with TIFNs. Li et al. (2010) investigated the values and ambiguities of TIFNs, thereby presented a method for MADM with TIFNs. Wu and Liu (2013) proposed a ranking method for IVTrIFNs based on the attitudinal score and accuracy expected functions. Wan and Dong (2014) developed a lexicographic ranking method for TIFNs based on the possibility mean and standard deviation.

The above researches seem to be effective to solve decision making on IFNs. However, there exist two main disadvantages. On the one hand, only the weighted arithmetic averaging operator has been defined for TIFNs, the other aggregation operators, such as the weighted geometric averaging operator, the ordered weighted aggregation operator and hybrid aggregation operator, have not been found in the existing literature. These aggregation operators can be employed to effectively integrate the decision information during the process of decision making (Merigó & Casanovas, 2010a, 2010b; Merigó, Casanovas, & Xu, 2014; Wei & Merigó, 2012; Zeng, Wang, Merigó, & Pan, 2014). On the other hand, existing ranking methods of TIFNs in (Li, 2010; Li et al., 2010; Nan et al., 2010) neglected DM's risk attitude. In fact, Yager (1988) argued that there is no unique best approach to ranking fuzzy numbers. In the same way, there is no unique best approach to ranking TIFNs. It is more reasonable and important to incorporate DM's risk attitude to rank TIFNs.

Compared with the definition of IFS, TIFN makes the membership and non-membership no longer just relative to a fuzzy concept of "excellent" or "good" by adding the triangular fuzzy number. TIFNs can express different dimension decision information and reflect DM's information more accurately. For example, in venture investment selection problem, the profitability of a candidate may be evaluated as a TIFN ((6,7,9);0.6,0.2), which means that the assessment value of the candidate is between 6 and 9, the most possible value is about 7 with the maximum satisfaction degree 0.6, the minimum dissatisfaction degree 0.2, and the hesitation degree 0.2. Therefore, studying the generalized aggregation operators of TIFNs and applying to MAGDM field would have important theoretical value and practical significance.

To overcome these drawbacks, this paper firstly introduces the weighted possibility attitudinal expected values of TIFNs and presents a new risk attitudinal ranking method of TIFNs. Then, the triangular intuitionistic fuzzy ordered weighted averaging (TIFOWA) operator and ordered weighted geometric (TIFOWG) operator are defined. In particularly, we further develop some new generalized aggregation operators for TIFNs and then propose a new method for MAGDM with TIFNs. The main works and features of this paper are illuminated as follows:

(1) According to the possibility theory (Fullér & Majlender, 2003), we define the weighted possibility attitudinal expected values of TIFNs and thereby present a new risk attitudinal ranking method of TIFNs. This ranking method can sufficiently consider DM's risk attitude and make the results more consistent with real situations. The sensitivity analyses on attitudinal character parameter are also given.
(2) The TIFOWA and TIFOWG operators are firstly introduced. Then the triangular intuitionistic fuzzy generalized ordered weighted averaging (TIFGOWA) operator and generalized hybrid averaging (TIFGHWA) operator are further developed. The TIFGOWA and TIFGHWA operators unify most of aggregation operators of TIFNs. For instance, the TIFOWG and TIFOWA operators are the special cases of the TIFGOWA operator; The triangular intuitionistic fuzzy hybrid weighted averaging (TIFHWA) operator and hybrid weighted geometrical (TIFHWG) operator are the special cases of the TIFGHWA operator.
(3) Utilizing the TIFGHWA and TIFWA operators, we propose a new method for solving the MAGDM problems with TIFNs and incomplete weight information. In this method, DMs' weights are determined by Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) (Hwang & Yoon, 1981), and the weights of attributes are objectively derived through constructing a multi-objective programming model which is transformed into a linear goal programming model to solve.
(4) Since the TIFGHWA operator has different forms through choosing different function g, DMs can obtain different decision results by using the TIFGHWA operator, which greatly enhances the flexibility and agility of decision making method.

The remainder of this paper unfolds as follows. In Section 2, we define the weighted possibility attitudinal expected values of TIFNs and present a new risk attitudinal ranking method of TIFNs. Section 3 introduces some triangular intuitionistic fuzzy arithmetic and geometric aggregation operators. In Section 4, we develop the TIFGOWA and TIFGHWA operators and discuss some of their desirable properties in detail. In Section 5, a new method is proposed to solve MAGDM problems with TIFNs and incomplete weight information. An investment selection example and the comparison analyses are given in Section 6. The conclusions are made in the last section.

2. Triangular intuitionistic fuzzy numbers

In this section, we review the definition and operation laws of TIFNs and give the Hamming distance for TIFNs. Then, the weighted possibility attitudinal expected values of TIFNs are defined and thereby a new risk attitudinal ranking method of TIFNs is put forward.

2.1. Preliminaries of triangular intuitionistic fuzzy numbers

Definition 1 (Li, 2010)). A TIFN $\tilde{a} = ((a, a, \bar{a}); w_{\tilde{a}}, u_{\tilde{a}})$ is a special IFS on the real number set R, whose membership function and non-membership function are defined as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-\underline{a}}{a-\underline{a}} w_{\tilde{a}}, & \text{if } \underline{a} \leqslant x < a \\ w_{\tilde{a}}, & \text{if } x = a \\ \frac{\bar{a}-x}{\bar{a}-a} w_{\tilde{a}}, & \text{if } a < x \leqslant \bar{a} \\ 0, & \text{if } x < \underline{a} \text{ or } x > \bar{a} \end{cases}$$

and