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Optimal design of a synthetic chart for monitoring process dispersion with unknown in-control variance

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ABSTRACT

For a stable manufacturing process, quality problems are often caused by changes in process dispersion. Although there have been plenty of research on the monitoring of process dispersion, the existing studies of synthetic charts for monitoring process dispersion only focus on the upward shift monitoring. However, the decrease shift monitoring is also necessary and important. In this paper, a synthetic S^2 chart is proposed to simultaneously monitor both upward and downward shifts and it consists of a Shewhart-type two-sided S^2 sub-chart and a conforming run length sub-chart. In the known in-control variance case, the conforming run length sub-chart only needs a lower control limit and the proposed synthetic S^2 chart is shown to be average run length (ARL) unbiased. The effect of parameter estimation on the proposed synthetic S^2 chart is also investigated as it is an important issue especially in the real manufacturing processes. Considering that the in-control variance is usually unknown and needs to be estimated by Phase I samples in practice, a new synthetic S^2 chart in which conforming run length sub-chart also only needs a lower control limit is developed when the in-control variance is estimated. Furthermore, optimal designs for both known and unknown parameter cases are studied. The advantage of the proposed chart in performance is shown in the results of the comparison with the ARL-unbiased S^2 chart. Also, an example illustrates the construction and application procedure of this proposed chart.

1. Introduction

Quality improvement has become an increasingly important strategy in the manufacturing process and other processes, such as service process and health-related process (Woodall & Montgomery, 2014). Statistical Process Control (SPC) is a set of statistical techniques that can help improve the quality and performance of a process. The control chart is an effective on-line monitoring technique and is widely used in production lines in manufacturing processes.

With the development of technology, the quality of product becomes high in recent years. High-yield production lines are very common in the manufacturing industry so that control chart for high-yield process becomes an important and applicable research topic. Time-between-event (TBE) control chart has been proved to be effective and efficient in monitoring high-yield process. For variable TBE charts, researchers developed exponential CUSUM chart (Lucas, 1985; Vardeman & Ray, 1985), exponential EWMA chart (Gan, 1998) and exponential chart (Xie, Goh, & Ranjan, 2002; Zhang, Xie, & Goh, 2006). And for attribute TBE charts, cumulative count of conforming (CCC) chart has been widely studied (Kuralmani, Xie, Goh, & Gan, 2002; Ranjan, Xie, & Goh, 2003; Xie, Goh, & Kuralmani, 2000). In fact, some researchers utilize the name of conforming run length (CRL) instead of the name of CCC, but they have the same statistical meaning. Along with the research of CRL or CCC chart, Wu and Spedding (2000) firstly proposed the synthetic chart, which combines the Shewhart $\overline{X}$ chart and the CRL chart for detecting the shifts in the process mean. Calzada and Scariano (2001) studied the robustness of the synthetic $\overline{X}$ chart to non-normality. Davis and Woodall (2002) presented a Markov chain model of the synthetic chart and used it to evaluate the zero-state and steady-state average run length (ARL) performance. Sim (2003) discussed the combination of the $\overline{X}$ and CRL charts to monitor the shifts in the process mean when the quality characteristic follows gamma and exponential distributions. After that, the synthetic chart appeals much attention due to its efficiency and effectiveness. Recently, a lot of research have been done in the synthetic chart for monitoring the shift in process mean (e.g., Aparisi & De Luna, 2009; Bourke, 2008; Calzada & Scariano, 2013a, 2013b; Castagliola & Khoo, 2009; Khoo, Wu, & Atta, 2008; Khoo, Lee, Wu, Chen, & Castagliola, 2011; Khoo,

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Wong, Wu, & Castagliola, 2012; Machado & Costa, 2014; Scariano & Calzada, 2009; Wu, Ou, Castagliola, & Khoo, 2010; Yeong, Khoo, Zhang, & Castagliola, 2012; Yeong, Khoo, Lee, & Rahim, 2013).

Also, the synthetic chart is applied to monitor the shift in process dispersion. Chen and Huang (2005) and Huang and Chen (2005) developed the synthetic charts for monitoring the process dispersion with the sample standard deviations and sample ranges, respectively. Khilare and Shirke (2012) proposed the nonparametric synthetic chart which is a combination of the sign sub-chart and the CRL sub-chart for detecting the increases in the process variation. Rajmanya and Ghute (2013) considered a synthetic *D* chart combining the *D* chart and the CRL chart and it is used to detect shifts in the standard deviation of normally distributed process. However, those papers only consider the synthetic chart for increases in the process variation and the known in-control parameter case. In manufacturing processes, both process improvement and deterioration are of interest and both of them represent shifts in the manufacturing processes. Hawkins and Zamba (2005) stressed that proper control on process dispersion requires the capability to detect decreases as well as increases. Also, it is difficult to predict the direction of shifts in practice especially when practitioner has little knowledge about the process. So, the study of a chart which can detect both upward and downward shifts in the process dispersion becomes necessary and important. Along with these ideas, we extend the one-sided synthetic charts introduced by Chen and Huang (2005) and Huang and Chen (2005) and propose a two-sided synthetic S^2 chart to monitor the process dispersion.

Parameter estimation is an important issue for implementing control chart. And it has an influence on the performance of the control chart seriously due to the variability of the estimators (Jensen, Jones-Farmer, Champ, & Woodall, 2006; Psarakis, Vyniou, & Castagliola, 2013; Quesenberry, 1993; Woodall & Montgomery, 2014). The results in Section 3 show that the influence of estimated parameter for the proposed chart is serious. Besides, in manufacturing processes, the process parameters are rarely known and are often estimated by the Phase I data set. However, it is desirable to identify and remove assignable causes at the early stage and thus bring the process back into control at an earlier time. Therefore we consider the design of the two-sided synthetic chart with adjusted control limits based on small Phase I data set and this extension can help practitioner implement the control chart in a more applicable way.

CRL sub-chart should be designed with only one control limit and this kind of design makes CRL sub-chart convenient to use in practice. In order to ensure CRL sub-chart in our proposed chart have only one limit, the fraction nonconforming function of the S^2 sub-chart should be designed to reach its minimum when the process is in control. The design of the proposed S^2 sub-chart ensures that the CRL sub-chart only needs to find increases in the fraction nonconforming relative to the in-control case and thus the resulting CRL sub-chart only needs one lower control limit. Another advantage of our research is that the proposed synthetic S^2 chart is average run length (ARL) unbiased when the in-control parameter is known.

The rest of this paper is structured as follows. In Section 2, we introduce the synthetic S^2 chart with known parameter and compare the performance of the proposed synthetic S^2 chart with the ARL-unbiased S^2 chart proposed by Zhang, Bebbington, Lai, and Govindaraju (2005) in terms of the run length properties. In Section 3, we study the effect of parameter estimation on the proposed synthetic chart. Next, in Section 4, we propose a procedure to design the new synthetic S^2 chart with the desired in-control ARL when the in-control variance is estimated. The optimal design parameters are obtained and the performance comparison is conducted between the new synthetic S^2 chart and the ARL-unbiased S^2 chart with adjusted control limits developed by Guo and Wang (2014). Finally, we use an example to illustrate the operation of the proposed synthetic S^2 chart.

2. Synthetic S^2 chart with known in-control variance

Similar to the synthetic $\overline{X}$ chart, the proposed synthetic S^2 chart consists of two sub-charts: a S^2 chart and a CRL chart.

2.1. S^2 chart

Suppose that $\{X_1, X_2, \ldots, X_n\}$ is a random sample of size n from the normal $N(\mu, \sigma_1^2)$ distribution, where $\sigma_1^2 = \rho\sigma_0^2$, μ is the process mean, σ_0 is the in-control standard deviation and $\rho > 0$ is the magnitude of the process variance shift. If $\rho = 1$, the process variance is in control, otherwise the process variance has shifted, namely, the process variance has decreased when $0 < \rho < 1$ or increased when $\rho > 1$.

Let $T(\sigma_0) = (n-1)S^2/\sigma_0^2$, where $\overline{X} = \sum_{j=1}^{n} X_j/n$, $S^2 = \sum_{j=1}^{n}(X_j - \overline{X})^2/(n-1)$. Notice that $T(\sigma_0) = \rho T(\sigma_1)$ and $T(\sigma_0)$ is an increasing function of ρ, thus $T(\sigma_0)$ has been proposed as a monitoring statistic (Montgomery, 2005; Guo & Wang, 2014; Zhang et al., 2005).

Let UCL and LCL be the upper and lower control limits of the S^2 sub-chart based on the monitoring statistic $T(\sigma_0)$, respectively. If a sample value of the monitoring statistic $T(\sigma_0)$ falls between the control limits of the S^2 sub-chart, the sample is regarded as a conforming sample, otherwise the sample is called a nonconforming sample. Thus, when the process variance is $\sigma_1^2 = \rho\sigma_0^2$, the fraction nonconforming is given by

$$\begin{aligned} p(\rho) &= 1 - P(LCL \leqslant T(\sigma_0) \leqslant UCL \mid \sigma^2 = \sigma_1^2) \\ &= 1 - P(LCL/\rho \leqslant T(\sigma_1) \leqslant UCL/\rho \mid \sigma^2 = \sigma_1^2) \\ &= 1 - F_{\chi^2_{n-1}}(UCL/\rho) + F_{\chi^2_{n-1}}(LCL/\rho), \end{aligned} \tag{1}$$

where $F_{\chi^2_k}(x)$ is the cumulative distribution function (c.d.f.) of the χ^2 distribution with k degrees of freedom.

2.2. Conforming run length chart

The conforming run length (CRL) chart proposed by Bourke (1991) was originally developed as attribute control chart to detect the shifts in the fraction nonconforming. The random variable *CRL* (i.e., the conforming run length) is defined as the number of conforming samples between two consecutive nonconforming samples (including the ending nonconforming sample). The idea of the CRL chart is that the mean of *CRL* is a decreasing function of the fraction nonconforming $p(\rho)$. If a sample value of *CRL* is large, it may indicate a decrease in the fraction nonconforming. On the other hand, if a sample value of *CRL* is small, it is an indication of an increase in the fraction nonconforming. Because the random variable *CRL* follows the geometric distribution with parameter $p(\rho)$, the mean and c.d.f. of *CRL* are given by

$$\mu_{CRL} = \frac{1}{p(\rho)} \tag{2}$$

and

$$F_{CRL}(x) = P(CRL \leqslant x) = 1 - (1 - p(\rho))^x, \quad x = 1, 2, \ldots, \tag{3}$$

respectively.