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Methods for ranking intuitionistic multiplicative numbers by distance measures in decision making ☆

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ABSTRACT

Intuitionistic multiplicative number (IMN) is the basic component of intuitionistic multiplicative set (IMS) and intuitionistic multiplicative preference relation, which is suitable for describing the preference information in the unbalance distribution. In this paper, we propose two methods of ranking IMNs by distance measures for applications in decision making. To do it, we first develop the normalized and weighted normalized Manhattan distances between IMSs by analyzing the correlations of IMSs and intuitionistic fuzzy sets quantitatively. As a specific case, we also present the normalized Manhattan distance between IMNs, based on which, we introduce the distance and accuracy functions of IMNs and give a comparison law for them. After that, we propose a method for ranking IMNs in decision making. Furthermore, considering the characteristics of the distance measures, we extend the distance and accuracy functions into IMSs and give the extended distance and accuracy functions of IMSs. Then we apply them to the decision making, especially group decision making, and propose another ranking method for IMNs based on an extended comparison law. Our methods can not only overcome the shortages of the existing method that uses the score and accuracy functions for ranking IMNs which may occur contradiction with our intuition, but also reduce the amount and time of computation.

1. Introduction

Intuitionistic multiplicative numbers (IMNs) are the basic components of the intuitionistic multiplicative set (IMS) (Xia, Xu, & Liao, 2013). The preference information included in IMNs contains three parts: the membership, non-membership and hesitation information about alternatives (or objects) given by the decision maker who utilizes the 1/9–9 scale (also called Saaty's 1–9 scale) (Jiang & Xu, 2014; Jiang, Xu, & Yu, 2013; Saaty, 1977; Xia & Xu, 2013; Xia et al., 2013; Xu, 2013; Yu & Fang, 2014), instead of the 0–1 scale (Atanassov, 1986; Atanassov & Gargov, 1989; Merigó & Casanovas, 2011; Xu, Wang, Sun, & Yu, 2014; Xu, 2007; Zhao, Xu, Liu, & Wang, 2012), to express his/her preference information which assumes that the grades between "fully accept" and "fully reject" are not distributed uniformly and symmetrically (see Table 1). In this way, IMNs can more effectively depict the unbalanced distribution that appears everywhere in actual life, and can solve some cases that are inconsistent with intuition by using intuitionistic fuzzy information (Jiang & Xu, 2014; Xia et al., 2013; Yu & Xu, 2014). One example is the law of diminishing marginal utility in economics (Jiang et al., 2013; Xia et al., 2013). When increasing the same consumption investment, a company with worse performance yields more utility than that with better performance. Another example is the rain attenuation prediction for satellite communication, which cannot be ignored under different frequencies, especially higher frequencies. According to the prediction model in ITU-R (International Telecommunication Union-Radio Communication Sector) (ITU-R Recommunication), with improving a certain frequency, it increases more rain attenuation in the higher operating frequencies than in the lower frequencies, and causes more cost to compensate the signal outage. That is to say, sometimes the gap between the grades expressing good information should be larger than the one between the grades reflecting bad information. In such situations, people prefer to express their preference information or judgments with unsymmetrical grades (Chiclana, Herrera, & Herrera-Viedma, 1998; Chiclana, Herrera, & Herrera-Viedma, 2001; Herrera, Herrera-Viedma, & Martínez, 2008; Jiang et al., 2013; Xia & Xu, 2013; Xia et al., 2013; Xu, 2013) due to the complex characteristics of practical problems.

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Table 1
The Saaty's 1–9 scale.

1–9 scale	Meaning
1/9	Extremely not accepted
1/7	Very strongly not accepted
1/5	Strongly not accepted
1/3	Moderately not accepted
1	Equally accepted
3	Moderately accepted
5	Strongly accepted
7	Very strongly accepted
9	Extremely accepted
Other values between 1/9 and 9	Intermediate values used to present compromise

Furthermore, IMNs are the basic and important components of the intuitionistic multiplicative preference relation (IMPR) (Xia et al., 2013) which is a useful tool for expressing the decision maker's preferences by comparing alternatives in pairs (Jiang et al., 2013; Xia & Xu, 2013; Xia et al., 2013; Xu, 2013; Yu & Fang, 2014;). Recently, some researchers have investigated IMNs and IMPRs, for example, Xia et al. (2013) introduced the concept of IMPR and developed some aggregation operators of IMNs for decision making; Yu and Fang (2014) also developed some aggregation operators of IMNs and applied them into decision making based on algebraic operational laws; Xia and Xu (2013) introduced some different intuitionistic multiplicative aggregation operators based on Choquet integral and power average, and then applied them to group decision making with IMPRs; Xu (2013) developed a method to derive the priority weights of the objects from an IMPR; Jiang et al. (2013) defined the compatibility degree for IMNs and applied it into the consensus reaching process for group decision making. In addition, Yu and Xu (2014) introduced the intuitionistic multiplicative triangular number as a generalization of the IMN. In addition, how to compare and rank the IMNs is a key or an unavoidable question in the decision making fields with IMNs. All studies mentioned above use the score and accuracy functions proposed in Xia et al. (2013) which is the unique existing method for ranking IMNs. The advantage of this ranking method is that it is of total order which can compare any two IMNs, but in practical applications, it sometimes contradicts with our intuition. How to avoid this issue is an interesting research topic, which is the focus of this paper.

It is known that the distance measure is used to describe the differences between two numbers (points or sets) and it can be considered as a dual concept of similarity measure (Wang & Xin, 2005). Due to the characteristics of distance measures, it has been successfully utilized to rank two intuitionistic fuzzy numbers (IFNs), fuzzy numbers and interval fuzzy numbers in decision making: Xu and Yager (2008) gave an approach for ranking IFNs by calculating the distances from the IFNs to the positive and negative ideal points; Szmidt and Kacprzyk (2009) improved this approach by taking into account both the distance between IFNs and the positive ideal point and the reliability of the IFNs; Zhang and Xu (2012) introduced another method by using the accuracy degree and a similarity function which is based on distance measures. Merigó and Casanovas (2011) presented a decision making approach by introducing the induced ordered weighted averaging distance operator based on distance measures and the induced aggregation operators. Xu et al. (2014) proposed a distance based aggregation approach to assess the relative importance weights for GDM with interval preference orderings. Inspired by it, we develop some distance measures between IMSs and IMNs in this paper and then propose two novel methods for ranking IMNs in the decision making.

The rest of the paper is arranged as follows: Section 2 reviews some basic knowledge. Section 3 develops some distance measures between IMSs and IMNs by analyzing the correlations of IMSs and IFSs quantitatively. In Section 4, based on the normalized Manhattan distance of IMNs, we introduce two functions of IMNs: the distance and accuracy functions, and give a comparison law for IMNs. After that, we propose a method for ranking IMNs in decision making. In Section 5, we introduce the extended distance and accuracy functions of IMSs, based on which we propose another method for ranking IMNs via an extended comparison law for IMSs in decision making which can greatly promote the computational efficiency, especially in group decision making. A numerical example is provided to illustrate our methods, and some comparisons on our methods and the previous work are presented in Section 6. Concluding remarks are given in Section 7.

2. Basic concepts

Definition 2.1 Xia et al. (2013). Let X be fixed, an intuitionistic multiplicative set (IMS) is defined as:

$$D = \{\langle x,\ \rho_D(x),\ \sigma_D(x)\rangle | x \in X\} \tag{1}$$

which assigns to each element x a membership information $\rho_D(x)$ and a non-membership information $\sigma_D(x)$, with the conditions: $1/9 \leqslant \rho_D(x),\ \sigma_D(x) \leqslant 9$, and $0 < \rho_D(x)\sigma_D(x) \leqslant 1, \forall x \in X$.

Xia et al. (2013) denoted the pair $(\rho_D(x), \sigma_D(x))$ as an IMN of x. For each IMS D in $X, \tau_D(x) = 1/\rho_D(x)\sigma_D(x)$ can be interpreted as the uncertain or hesitant information. Obviously, $1 \leqslant \tau_D(x) \leqslant 9^2, \forall x \in X$. Then an IMN can also be represented by $(\rho_D(x),\ \sigma_D(x), \tau_D(x))$.

To compare any two IMNs, Xia et al. (2013) defined the following comparison laws:

Definition 2.2 Xia et al. (2013). For an IMN $\alpha = (\rho_\alpha, \sigma_\alpha)$, we call $s(\alpha) = \rho_\alpha/\sigma_\alpha$ the score function of α, and $r(\alpha) = \rho_\alpha\sigma_\alpha$ the accuracy function of α. To compare two IMNs α_1 and α_2, we have

(1) If $s(\alpha_1) > s(\alpha_2)$, then $\alpha_1 > \alpha_2$;
(2) If $s(\alpha_1) = s(\alpha_2)$, then
  (a) If $r(\alpha_1) > r(\alpha_2)$, then $\alpha_1 > \alpha_2$;
  (b) If $r(\alpha_1) = r(\alpha_2)$, then $\alpha_1 = \alpha_2$.

By utilizing Definition 2.2, we can build some linear orders of IMNs (Xia et al., 2013). For example,

Example 1. Let

$$\alpha_1 = \left(\frac{1}{8}, \frac{1}{4}, 32\right), \quad \alpha_2 = \left(\frac{1}{6}, \frac{1}{3}, 18\right), \quad \alpha_3 = \left(\frac{1}{4}, \frac{1}{2}, 8\right),$$
$$\alpha_3 = \left(\frac{1}{2}, 1, 2\right)$$

be four IMNs, then we can calculate their score and accuracy values, respectively:

$$s(\alpha_1) = s(\alpha_2) = s(\alpha_3) = s(\alpha_4) = \frac{1}{2}$$
$$r(\alpha_1) = \frac{1}{32}, \quad r(\alpha_2) = \frac{1}{18}, \quad r(\alpha_3) = \frac{1}{8}, \quad r(\alpha_4) = \frac{1}{2}.$$

Then we have $\alpha_4 > \alpha_3 > \alpha_2 > \alpha_1$.

However, this ranking method has some flaws which can be illustrated in the following example: