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Ordered visibility graph weighted averaging aggregation operator: A methodology based on network analysis

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A B S T R A C T

The ordered weighted averaging (OWA) operators are widely used in many applications. Determining the OWA operator weights is still an open issue. In this paper, an ordered visibility graph weighted averaging (OVGWA) aggregation operator is proposed based on complex network methodology. After the argument values are ordered in descending order, a network is constructed using ordered visible graph (OVG) method. The arriving probabilities of random walker in the network are utilized as the weights of the corresponding data. Compared with the classical OWA, the proposed OVGWA not only takes the information of orders but also the argument values themselves into consideration to determine the weights. An application of produced water management is used to illustrate the application of the proposed OVGWA aggregation operator. In addition, the aggregated results are compared with those obtained by maximal entropy method. The results show that the proposed method is effective. To study the impacts of the influencing factors, a sensitivity analysis has been carried out. It is utility to understand the role of different indicators on aggregated value.

1. Introduction

The ordered weighted averaging (OWA) operator was first introduced by Yager (1988), which was widely used in many fields such as expert systems (Fullér & Majlender, 2001; Islam et al., 2013; Yager, 2010b), decision making (Sadiq, Rodríguez, & Tesfamariam, 2010; Sadiq & Tesfamariam, 2007; Tesfamariam, Rajani, & Sadiq, 2006; Tesfamariam, Sadiq, & Najjaran, 2010; Yager, 2010a), environment assessment (Sadiq, Haji, Cool, & Rodriguez, 2010; Sadiq & Tesfamariam, 2009; Yeheyis, Hewage, Alam, Eskicioglu, & Sadiq, 2013; Zhang, Achari, Sadiq, Langford, & Dore, 2012) and risk analysis (Deng, Sadiq, Jiang, & Tesfamariam, 2011; Sadiq, Kleiner, & Rajani, 2007; Sadiq & Tesfamariam, 2008). It provided a unified framework for decision making under uncertainly environment where different decision criteria such as optimistic, pessimistic and equally likely were characterized by different OWA operator weights (Ma, Xiong, & Luo, 2013; Xia & Xu, 2011; Xia, Xu, & Chen, 2013; Xu & Yager, 2010; Yager & Alajlan, 2013). To address decision making issues under uncertain environment, fuzzy sets theory (Jiang, Luo, Qin & Zhan, 2015a; Jiang, Yang, Luo & Qin, 2015b; Kahraman, Onar & Oztaysi, 2015; Liu, Liu & Lin, 2013a; Liu, 2014; Rikhtegar et al., 2014; Zadeh, 1965) and evidence theory (Bandyopadhyay & Bhattacharya, 2015; Dempster, 1967; Deng, Hu, Deng & Mahadevan, 2014; Deng, Mahadevan & Zhou, 2015; Deng, 2015; Fu & Yang, 2012; Kabir, Tesfamariam, Francisque & Sadiq, 2015; Liang, Pedrycz, Liu & Hu, 2015; Liu, Pan & Dezert, 2013b; Liu, Pan, Dezert & Mercier, 2014; Shafer, 1976; Su, Mahadevan, Xu & Deng, 2015; Wang, Dai, Chen & Meng, 2015) are widely used to model uncertain information. As a result, many OWA operators combined with fuzzy set theory and evidence theory are presented Yang and Pang (2014) and Reformat and Yager (2007). Generally speaking, OWA will play a more and more important role in decision making field. The determination of the associated weights is a very crucial issue in the application of OWA aggregation. A lots of studies have been carried out on obtaining the weights (Chen, Hu, Mahadevan, & Deng, 2014; Xiong & Liu, 2014; Yager, 2010c; Zeng, Merigó, & Su, 2013). One of the important methodologies for obtaining the

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weights was proposed by O'Hagan (1988). A maximum entropy method was presented in O'Hagan (1988), which formulated the OWA operator weights problem as a constrained nonlinear optimization model. This model needed a predefined degree of *orness* as its constraint and the entropy as its objective function. The weights were referred to as the maximum entropy weights. In other words, the maximum entropy method determines a special type of OWA operators with maximum entropy of the OWA weights for a given level of *orness*. The method of Lagrange multipliers was used by Fullér and Majlender (2001) to solve O'Hagan's procedure analytically.

The OWA operator extended to different kinds of versions. Three of the most basic extensions were widely investigated: the induced OWA (IOWA), the uncertain OWA (UOWA), and the generalized OWA (GOWA). The IOWA operator (Yager & Filev, 1999) was a practical extension of OWA. Different from original OWA, IOWA used order-inducing variables in the reordering of the arguments. Its main advantage was that it could represent more complex situations since it could include a wide range of factors in the reordering process rather than simply considered the values of the arguments. The UOWA operator was investigated by Xu and Da (2002). In UOWA operator, the associated weighting parameters could not be specified, but value ranges could be obtained and each input argument was given in the form of an interval of numerical values. It was a very useful operator for uncertain situations where the information cannot be assessed with exact numbers. In 2004, Yager (2004) presented the GOWA operators. These operators added to the OWA operator an additional parameter controlling the power to which the argument values were raised. Recently, based on the above three extensions, some further promotions and combination were investigated. In 2009, Merigó and Gil-Lafuente (2009) presented the induced generalized ordered weighted averaging (IGOWA) operator which included the main characteristics of both the generalized OWA and the induced OWA operator. This operator used generalized means and order-inducing variables in the reordering process. It provided a very general formulation that included as special cases a wide range of aggregation operators. After that, Merigó and Casanovas (2010) studied the GOWA operator under uncertain situations where the available information was given in the form of fuzzy numbers and the fuzzy generalized ordered weighted averaging (FGOWA) operator was presented. They (2011b) also investigated the uncertain generalized OWA (UGOWA) operator where the uncertain information represented as interval numbers. Afterwards, other versions of IOWA operators under uncertain situations (Merigó & Casanovas, 2011a, 2011c; Merigó & Gil-Lafuente, 2013; Merigó, Gil-Lafuente, Zhou, & Chen, 2012) and other extensions of OWA (Merigó, 2011; Merigo & Yager, 2013; Zhou & Chen, 2011) were further improved.

It should be pointed out that the existing methods are complex to some degree and it's hard to avoid subjective factors once the parameters required, to determine the weights in OWA. It is necessary to develop a new and simple method for the real application. In this paper, we focus on the issue how to determine the weights in a more objective and reasonable way. We first develop an ordered visible graph method which converts the ordered data into a network. Then, a random walks model is employed to calculate the arriving probability of each node. That is to say, the weights are determined by the arriving probabilities in the ordered visible graph.

This paper is organized as follows: In Section 2, we briefly introduce OWA operator and the visible graph method. In Section 3, our proposed ordered visibility graph weighted averaging (OVGWA) aggregation operator is detailed. In Section 4, an application of produced water management is used to illustrate the proposed method. Finally, some conclusions are given in Section 5.

2. Preliminaries

In this section, the OWA operator and the visibility graph are briefly introduced.

2.1. The OWA operator and its weight generation method

An OWA operator (Yager, 1988) of dimension n is a mapping:

$$F : \Re^n \rightarrow \Re,$$

that has an associated weighting vector W:

$$W = [w_1 w_2 \dots w_n]^T$$

such as that

$$\sum_i w_i = 1; \quad w_i \in [0, 1]$$

and where

$$f(a_1, \dots, a_n) = \sum_{j=1}^{n} w_j b_j,$$

where b_j is the jth largest element of the collection of the aggregated objects $\{a_1, a_2, \dots, a_n\}$.

A fundamental aspect of this operator is the re-ordering step, in particular an aggregated a_i is not associated with a particular weight w_i but rather than a weight is associated with a particular ordered position of the arguments.

One key issue in the OWA operator is to determine its associated weights. In Yager (1988), two characterizing measures associated with weighting vector W of an OWA operator were introduced by Yager. The first one, the measure of *orness* of the aggregation, is defined as

$$orness(W) = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i.$$

and it characterizes the degree to which the aggregation is like an *or* operation. It is clear that $orness(W) \in [0, 1]$ holds for any weighting vector.

The second one, the measure of *dispersion* of the aggregation, is defined as

$$disp(W) = -\sum_{i=1}^{n} w_i ln w_i$$

and it measures the degree to which W takes into account all information in the aggregation.

In order to determine OWA operator weights, these two measures to generate the OWA operator were used by O'Hagan (1988). A maximum entropy method was suggested by him, which required the solution of the following constrained nonlinear optimization model:

$$\begin{aligned} &Maximize \quad Disp(W) = -\sum_{i=1}^{n} w_i ln w_i \\ &s.t. \quad orness(W) = \alpha = \frac{1}{n-1} \sum_{i=1}^{n} (n-i) w_i, \quad 0 \leqslant \alpha < 1, \\ &\sum_{i=1}^{n} w_i = 1, \quad 0 \leqslant w_i \leqslant, \quad i = 1, \dots, n. \end{aligned} \tag{1}$$

The problem (1) was well addressed based on Lagrange multipliers method by Fullér and Majlender (2001). The main calculation process is as follows: