Computers & Industrial Engineering 88 (2015) 229–236

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Profit maximization of TSP through a hybrid algorithm

Aditi Khanra [a], Manas Kumar Maiti [b,*], Manoranjan Maiti [c,*]

[a] Bohichberia High School (H.S.), Bohichberia, Purba Medinipur, West Bengal 721649, India
[b] Department of Mathematics, Mahishadal Raj College, Mahishadal, Purba-Medinipur, WB 721628, India
[c] Department of Applied Mathematics with Oceanology and Computer Programming, Vidyasagar University, Paschim-Medinipur, WB, India

ARTICLE INFO

Article history:
Received 24 November 2014
Received in revised form 12 June 2015
Accepted 24 June 2015
Available online 15 July 2015

Keywords:
Ant colony optimization
Particle swarm optimization
Travel cost
Travel time
Profit
Hybrid algorithm

ABSTRACT

Here a new model of Traveling Salesman Problem (TSP) with uncertain parameters is formulated and solved using a hybrid algorithm. For this TSP, there are some fixed number of cities and the costs and time durations for traveling from one city to another are known. Here a Traveling Salesman (TS) visits and spends some time in each city for selling the company's product. The return and expenditure at each city are dependent on the time spent by the TS at that city and these are given in functional forms of t. The total time limit for the entire tour is fixed and known. Now, the problem for the TS is to identify a tour program and also to determine the stay time at each city so that total profit out of the system is maximum. Here the model is solved by a hybrid method combining the Particle Swarm Optimization (PSO) and Ant Colony Optimization (ACO). The problem is divided into two subproblems where ACO and PSO are used successively iteratively in a generation using one's result for the other. Numerical experiments are performed to illustrate the models. Some behavioral studies of the models and convergences of the proposed hybrid algorithm with respect to iteration numbers and cost matrix sizes are presented.

1. Introduction

The TSP (Applegate, Bixby, Chvtal, & Cook, 2007) is one of the most widely studied NP-hard combinatorial optimization problem which cannot be solved exactly in polynomial time. It is also an important research topic. This problem is, for a given set of n cities with travel costs (or distances) c_{ij} between each pair of cities $i,j \in [1:n]$, to determine a minimum cost (or distance) circuit (Hamiltonian circuit or cycle) passing through each vertex once and only once. Every such tour together with a start city can be characterized by the permutation of all cities as they are visited along the tour. TSP has important applications to real world problems, such as vehicle routing problem, mixed chinese postman problems, and printed circuit board punching sequence problems.

Different types of TSPs have been solved by the researchers during last two decades. Among these, TSP with time windows (Focacci, Lodi, & Milano, 2002), Stochastic TSP (Chang, Wan, & OOI, 2009; Liu, 2010), Double TSP (Petersen & Madsen, 2009), Asymmetric TSP (Majumdar & Bhunia, 2011), TSP with precedence constraints (Moon, Ki, Choi, & Seo, 2002), Probabilistic TSP (PTSP) (Bianchi, Gambardella, & Dorigo, 2002), etc. are worth mentioning. In TSP with precedence constraints, there exists an order in which the cities should be visited. In Asymmetric TSP, cost of traveling from vertex (node/city) v_i to v_j is not equal to the cost of traveling from vertex v_j to v_i. In Stochastic TSP, each vertex is visited with a given probability and goal is to minimize the expected distance/cost of a priori tour. In TSP with time windows, each vertex is visited within a specified time windows. PTSP is a TSP problem where each customer has a given probability of requiring a visit.

During the last decades, several algorithms emerged to approximate the optimal solution of TSP such as Tabu search method (Fiechter, 1994), Neural networks (Leung, Jin, & Xu, 2004; Masutti & Castro, 2009), Simulated annealing (Lo & Hus, 1998), Genetic algorithms (GA) (Cheng & Gen, 1994; Cheng, Gen, & Sasaki, 1995; Liu, 2010; Majumdar & Bhunia, 2011; Nagata & Soler, 2012; Yang, Wu, Lee, & Liang, 2008), Ant Colony Optimization (ACO) (Bai, Yang, Chen, Hu, & Pan, 2013; Cheng & Mao, 2007; Dorigo & Gambardlla, 1997; Ghafurian & Javadian, 2011; Ibanez & Blum, 2010), and Particle Swarm Optimization (PSO) (Chen & Chien, 2011; Lin, Chen, & Lin, 2009).

ACO is an important soft computing technique for solving optimization problems. In ACO, the behavior of real ants to find the shortest path between their nest and food sources, has been used. Several ACO algorithms are available to solve the well-known

* Corresponding authors.
E-mail addresses: aditikhanra@gmail.com (A. Khanra), manasmaiti@yahoo.co.in (M.K. Maiti), mmaiti2005@yahoo.co.in (M. Maiti).

http://dx.doi.org/10.1016/j.cie.2015.06.018
0360-8352/

NP-Hard TSP. Dorigo and Gambardlla (1997) described an artificial ant colony capable of solving the TSP. In 2007, Cheng and Mao presented a modified ant colony system for solving the TSP with time windows. Ibanez and Blum proposed a Beam-ACO which is a hybrid method combining ACO with beam search to solve TSP in 2010. In 2011, Ghafurian and Javadian proposed an ACO algorithm for solving fixed destination multi-depot multiple traveling salesmen problems. In 2013, Bai et al. and Pan proposed a model inducing max–min ant colony optimization for Asymmetric TSP.

PSO is a heuristic optimization technique based on swarm intelligence that is inspired by the behavior of birds flocking. It was first proposed by Kennedy and Eberhart (Eberhart & Kennedy, 1995; Kennedy & Eberhart, 1995) and has received significant attention. Many studies modified the PSO algorithm to improve its performance in continuous optimization. Among them, one important improvement was adding the inertia weights into the updating rules of PSO (Shi & Eberhart, 1998). Ratnaweera, Halgamuge, and Watson (2004) used time-varying strategies of inertia weights. Later, Clerc and Kennedy (2002) presented a construction factor to form a new inertia weights, which has proven effective and is the current state of the art. A large number of different algorithms have been developed to find suboptimal solutions for the TSP in polynomial time.

Inspite of the above developments, there are some lacunas/gaps in forming the realistic TSPs, These are:

- Though the visit of a TS is organized in order to get a return for the company, till now, none has considered the returns at each city out of the tour.
- Most of the TSPs are concerned with the minimization of tour cost or travel time. But, for selling or canvassing a product, a TS has to spend some time at each city and incurs some expenditure for this. This also has been overlooked by the TSP researchers.
- Normally, a TS is asked by his/her company to finish the entire activities including the tour(travel and stay times) within a specified time limit. This constraint has been taken into account by very few investigators only.
- Though ultimate goal of a company is to make the profit through the sales representative, till now, no TSP has been formulated as a profit maximization problem considering returns and expenditures.
- In order to bridge the above gaps, one new TSP has been formulated and solved.

The above assumptions are realistic and have applications in medicine producing firms with respect to their medical representatives and salesmen. For a medical firm, normally a medical representative/salesman is asked to tour a number of towns/sub-towns within a limited time. In this process, he/she fixes the paths and spends some times at the towns/stations in such way that total outcome/benefit of the tour is maximum. He/she incurs some costs for travel, pays for stay at stations and earns indirectly through Doctors/Medical shops by canvassing and presentation. All these are done within the limited time fixed by the medical firm. The proposed TSP incorporating the above assumptions are most appropriate for the real life medical house problems and incorporated in the proposed model.

In this paper, for the proposed TSP, a TS visits some fixed number of cities following the TSP rule and spends sometime at each visited city. He/she earns some returns and incurs some expenditure at each city and these are stay time dependent in some functional forms of time. Here total allowable time for the entire tour including stay times is fixed. Now, the problem for the TS is to fix the tour program and the stay times at each city so that total profit out of the tour is maximum. Thus TSP is formulated as maximization problem with deterministic returns and stay expenditures.

In comparison to the usual TSP, here the proposed TSP consists of two sub-optimization models – (i) minimization of total travel costs between the cities and (ii) allocation of stay times at the cities so that total outcome due to stay is maximum. Then a trade off between these two sub-models is made. For these two fold TSP optimization problem, a hybrid algorithm combining the algorithms of ACO and PSO is designed and applied successfully. Here ACO and PSO are used successively and iteratively in a generation using one's result for the other. The proposed TSP is illustrated with numerical examples. Some interesting model behaviors are presented with different sizes of cost matrices. It is shown that more stay at stations does not fetch more profit. Some parametric studies of the proposed hybrid algorithm with respect to iteration numbers and cost matrices sizes are also presented.

Rest of the paper is organized as follows. Models are formulated in Section 2. In Section 3, Hybrid ACO–PSO system is described. In Section 4, experimental results are presented. Finally a brief discussion, models' behavioral and different sensitivity studies and conclusions are respectively drawn in Sections 5–7.

2. Model formulation

2.1. Classical TSP for minimum total travel cost (Model1A)

In a classical two-dimensional TSP, TSP can be represented as graph $G = (V, E)$, where $V = 1, 2, \ldots, N$ is the set of nodes and E is the set of edges. A salesman has to travel N cities at minimum cost. In this tour, salesman starts from a city, visits all the cities exactly once and comes to the starting city using minimum cost. Let c_{ij} be the cost for traveling from i-th city to j-th city. Then the model is mathematically formulated as (Dantzig, Fulkerson, & Johnson, 1954):

$$\left.\begin{array}{ll} \text{Determine} & x_{ij}, \quad i = 1,2,\ldots N, \quad j = 1,2,\ldots N. \\ \text{to minimize} & Z = \sum_{i=1}^{N}\sum_{j=1}^{N} x_{ij}c_{ij}, \\ \text{subject to} & \sum_{i=1}^{N} x_{ij} = 1, \quad j = 1,2,\ldots,N \\ & \sum_{j=1}^{N} x_{ij} = 1, \quad i = 1,2,\ldots,N \\ & \sum_{i\in S}\sum_{j\in S} x_{ij} \leqslant |S| - 1, \quad \forall S \subset V, \quad x_{ij} \in \{0,1\}. \end{array}\right\} \tag{1}$$

where $x_{ij} = 1$ if the salesman travels from city-i to city-j, otherwise $x_{ij} = 0$.

Let $(x_1, x_2, \ldots, x_N, x_1)$ be a complete tour of a salesman, where $x_i \in \{1, 2, \ldots, N\}$ for $i = 1, 2, \ldots, N$ and all x_i's are distinct. Then the above model reduces to

$$\left.\begin{array}{l} \text{Determine a complete tour } (x_1, x_2, \ldots, x_N, x_1) \\ \text{to minimize } Z = \sum_{i=1}^{N-1} c_{x_i,x_{i+1}} + c_{x_N,x_1} \end{array}\right\} \tag{2}$$

Classical TSP with time for minimum total travel time (Model1B): Let t_{ij} be the time for traveling from i-th city to j-th city. In this case, the model formulation is the same as (1) except