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An integrated inventory model with capacity constraint and order-size dependent trade credit

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A B S T R A C T

Trade credit has many forms in today's business practice. The most common form of trade credit policy that is used to encourage retailers to buy larger quantities is order-size dependent. When the number of ordered units exceeds the capacity of the own warehouse, an additional rented warehouse is required to store the excess units. Therefore, to incorporate the concept of order-size dependent trade credit and limited storage capacity, we proposed an integrated inventory model with capacity constraint and a permissible delay payment period that is order-size dependent. In addition, the unit production cost, which is a function of the production rate, is considered. Three theorems and an algorithm are developed to determine the optimal production and replenishment policies for both the supplier and the retailer. Finally, numerical examples are presented to illustrate the solution procedure and the sensitivity analyses of some key parameters are provided to demonstrate the proposed model.

1. Introduction

A traditional EOQ model makes a basic assumption: the retailer needs to pay the full amount to the supplier when the products are received. However, in real market transactions, retailers usually do not need to pay the total amount at the time the product is received; they are allowed delayed payments by suppliers instead. This type of trade credit is very common in today's business world. For suppliers, offering delayed payments may attract more retailers and thus increase sales. For retailers, not only does this lower the opportunity cost of capital, but it also allows them to earn interest on the revenue of goods sold within the permissible delay period. Hence, trade credit policy is beneficial to both suppliers and retailers. Goyal (1985) was the first to relax the assumption of the traditional EOQ model in which payment is made once products are received, and established an EOQ model based on a fixed delay payment period. Afterwards, many researchers proposed inventory models relating to the permissible delay in payments such as Aggarwal and Jaggi (1995), Shin (1997), Jamal, Saker, and Wang (2000), Salameh, Abboud, El-Kassar, and Ghattas (2003), Teng, Chang, and Goyal (2005), Cheng, Lou, Ouyang, and Chiang (2010), and Chung (2012). The above mentioned papers regarded the delayed payment period as a fixed value.

In real life situations, the trade credit policy that the supplier offers could have many variations. Some researchers assumed that the length of the delayed payment period is related to the retailer's order quantity, e.g., Chang, Ouyang, and Teng (2003), Shinn and Hwang (2003), Chang (2004), Chung, Goyal, and Huang (2005), Huang (2007a), Chung, Lin, and Srivastava (2012, 2013). In addition, some researchers assumed that both retailers and customers can buy with trade credit which is called a two-level trade credit policy. There were several relevant papers related to the two-level trade credit policy, such as Huang (2003, 2007b), Biskup, Simons, and Jahnke (2003), Jaggi, Aggarwal, and Goel (2007), Jaggi, Goyal, and Goel (2008), Teng and Goyal (2007), Liao (2008), Teng and Chang (2009), Min, Zhou, and Zhao (2010), Sharma, Goel, and Dua (2012), and Teng, Yang, and Chern (2013). Moreover, some researchers assumed that the supplier provides the delay payment and cash discount simultaneously, i.e., the supplier offers a cash discount to the retailer to encourage him to settle the account earlier. The available results adopting this assumption can be found in the work of Ouyang, Chen, and Chuang (2002), Ouyang, Teng, Chuang, and Chuang (2005), Chang (2002), Goyal, Teng, and Chang (2007), Sana and Chaudhuri (2008), and Yang (2010). All of the above mentioned papers discussed the issue from the

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Table 1
The comparison between this paper and previous studies with trade credit and limited storage space.

Papers	Integrated/EOQ	Trade credit policy	Limited storage space
Huang (2006)	EOQ	Two-level trade credit	Yes
Ouyang et al. (2007)	EOQ	Fixed credit period and cash discount for early payment	Yes
Liang and Zhou (2011)	EOQ	Fixed credit period	Yes
Zhou et al. (2012)	Two-echelon decentralized supply chain	fixed credit period	Yes
Liao et al. (2012)	EOQ	Conditional trade credit	Yes
Liao et al. (2013)	Integrated	Fixed credit period	Yes
This paper	Integrated	Order-size dependent trade credit	Yes

perspective of the supplier or the retailer, and just focused on one-sided optimal strategies.

With the impact of market globalization, businesses must integrate their supply chains to enhance their operational efficiency, respond to customers' needs more efficiently, and lower inventory costs. In a non-totalitarian supply chain system, since there is a latent difference in motives, the objectives of the supplier and the retailer may conflict with the objective of the entire supply chain. This will cause the optimal decision of the supplier or the retailer to not match that of the supply chain. When the activities and the decisions in a supply chain are inconsistent or not coordinated, the supply chain will lose its competitive advantages. Goyal (1976) first developed an integrated inventory model to determine the optimal joint inventory policy for a single supplier and a single retailer. Abad and Jaggi (2003) combined the concept of an integrated inventory model and trade credit policy, and established a supplier–retailer integrated system in which the supplier offers trade credit to the retailer. Afterward, several models concerning the integrated inventory model with various trade credit policies can be found in Jaber and Osman (2006), Yang and Wee (2006), Chen and Kang (2007, 2010), Su, Ouyang, Ho, and Chang (2007), Ho, Ouyang, and Su (2008), Ouyang, Ho, and Su (2008, 2009), Chang, Ho, Ouyang, and Su (2009), Thangam and Uthayakumar (2009), Huang, Tsai, Wu, and Chung (2010), Lin, Ouyang, and Dang (2012), Ouyang and Chang (2013), Chern, Pan, Teng, Chan, and Chen (2013), Chung, Lin, and Srivastava (2014), Chung, Lin, and Srivastava (2015), and their references. The above papers assumed that the warehouse owned by the retailer is adequate to store the entire procured inventory.

In practice, retailers might order more items when faced with an attractive price discount or when the ordering cost is relatively higher than the holding cost. If the space in the own warehouse (OW) is not sufficient to store all the purchased units, an external rented warehouse (RW) is used to hold the excess units. Usually, the RW has a higher unit holding cost than the OW; hence, purchased units fill up in the OW first and the excess units are kept in the RW. In addition, the units at the RW should be exhausted before units from the OW are retrieved. Hartley (1976) first established an inventory model with two levels of storage. This research topic attracted the attention of many researchers. Some related studies in this area include Pakkala and Achary (1992), Goyal and Giri (2001), Zhou and Yang (2005), Yang (2006), Dye, Ouyang, and Hsieh (2007), Hsieh, Dye, and Ouyang (2008), Rong, Mahapatra, and Maiti (2008), Lee and Hsu (2009), Panda, Maiti, and Maiti (2010), and their references. All of the above papers discuss a two-warehouse inventory problem with or without deterioration. However, the impact of trade credit policy on the optimal solution is not considered in these papers. Huang (2006) first established an EOQ model with two warehouses and a two-level trade credit policy. Ouyang, Wu, and Yang (2007) proposed an EOQ model with limited storage capacity and trade credit. The trade credit policy discussed in their paper is that the supplier not only provides a delayed payment period to the retailer, but also offers a cash discount if the retailer pays earlier. Later, Liang and Zhou (2011) developed a two-warehouse inventory model with trade credit policy for deteriorating items, which assumed that the RW has a higher deterioration rate than the OW. Recently, Liao, Huang, and Chung (2012) established an EOQ model for deteriorating items with two warehouses and the delay payment is permitted only when the ordering quantity meets a given threshold. These papers discussing two warehouses determined the optimal inventory policy only from the retailer's perspective.

Since trade credit policy is a common business practice nowadays, it is important to explore the impact of trade credit policy on an integrated inventory problem. Zhou, Zhong, and Li (2012) proposed a supplier-Stackelberg game inventory model with trade credit and limited storage space where the permissible delay period is a fixed given parameter. Liao, Chung, and Huang (2013) established an integrated inventory model for deteriorating items with trade credit and two warehouses. Likewise, the delay payment period in this study is a fixed given parameter. The studies that addressed the issues of limited storage space and trade credit policy simultaneously are shown in Table 1. When the length of the delayed payment is linked to order quantity rather than a given parameter, the retailer is encouraged to order large quantities with a longer delayed payment period so that the order quantity might exceed the OW capacity. As shown in Table 1, previous studies did not consider the impact of various delay payment length on order quantity. Therefore, this paper deals with an integrated inventory problem with an order-size dependent trade credit and limited storage capacity. In addition, the unit production cost in this study is considered as a function of the production rate. From theoretical analysis, three theorems are developed to determine the optimal policy for the supplier and the retailer. Finally, numerical examples are given to illustrate the solution procedure and a sensitivity analysis of the optimal solution with respect to the major parameters is performed.

2. Notation and assumptions

The following notation and assumptions are made in deriving the model:

Notation:

D	Retailer's demand rate
R	Supplier's production rate, $R > D$
A_R	Retailer's ordering cost per order
A_S	Supplier's setup cost per setup
r_{R1}	Retailer's holding cost rate for items in the OW, excluding interest charges
r_{R2}	Retailer's holding cost rate for items in the RW, excluding interest charges, $r_{R2} > r_{R1}$
r_S	Supplier's holding cost rate, excluding interest charged
F_0	Fixed transportation cost per shipment
F_1	Unit transportation cost
$c(R)$	Supplier's unit production cost which is a convex function of R
v	Retailer's unit purchasing cost, $v > c$