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Generalized compensative weighted averaging aggregation operators ☆

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ABSTRACT

The compensative weighted averaging (CWA) operator is generalized to develop a class of powerful generalized compensative weighted averaging (GCWA) operators with a very fine range of compensatory effects. The conventional means are shown to be the special cases of the proposed GCWA operator. A few extensions are investigated by combining GCWA operator with ordered weighted averaging (OWA) and induced OWA (IOWA) operators. An exponential class of aggregation operators such as exponential CWA, exponential OWA and exponential IOWA operators are developed. The usefulness of GCWA operators is shown through several examples and a case-study.

1. Introduction

The conventional t-norms and t-conorms are unable to model the compensation behaviour that inevitably characterizes any human aggregation process. The issue of compensative aggregation is investigated at length in Dyckhoff and Pedrycz (1984), Thole, Zimmermann, and Zysno (1979), Zysno (1979), Zimmermann and Zysno (1980), Luhandjula (1982), Krishnapuram and Lee (1992), Yager (1988), Aggarwal (2015). Generalized means is proposed in Dyckhoff and Pedrycz (1984) as a compensative aggregation operator. The compensative weighted averaging (CWA) operator (Aggarwal, 2015) is another compensative aggregation operator. Both CWA and generalized means have minimum and maximum as the limit cases, and the weighted averaging mean as a special case.

In this work, we combine CWA operator with the generalized means to create a family of aggregation operators with wide compensatory capabilities. We term the new class of the operators, so obtained, as generalized compensative weighted averaging (GCWA) operators. The proposed generalization provides an additional parameter that controls the power to which the arguments of aggregation are raised. This parameter together with the one originally present in CWA operators generate a vast range of compensative aggregation operators.

The varying compensatory effects are produced by different possible combinations of the two parameters, resulting into a continuum of aggregation values. The minimum and maximum of the argument values form the cornerstones of this continuum. As the special cases of the proposed GCWA operator are all the popular existing means. We investigate these special cases and the properties of GCWA operators in detail.

We also combine GCWA operators with ordered weighted averaging (OWA) (Yager, 1988) and induced OWA (IOWA) (Yager & Filev, 1999) operators to add to their capabilities. We term the resulting operators as generalized compensative ordered weighted averaging (GCOWA) and generalized compensative induced ordered weighted averaging (GCIOWA) operators. In contrast to OWA and IOWA operators, where the weight vector generates the compensation, parameters determine the compensation in the proposed GCOWA and GCIOWA operators.

The remainder of the paper is organized as following. Section 2 gives a brief background of the compensative weighted averaging operator. Section 3 is devoted to the proposed generalized compensative weighted averaging (GCWA) operator. In Section 4, we delve upon the limit and special cases of GCWA operator, and illustrate them through examples. Section 5 introduces the generalized compensative ordered weighted averaging (GCOWA) and generalized compensative induced ordered weighted averaging operators (GCIOWA). The special cases of GCOWA and GCIOWA operators are examined in detail. Section 6 gives an application of the proposed GCWA operator in a multi-criteria decision making case-study. Section 7 concludes the paper with an outlook on future work.

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2. Compensative weighted averaging operator

We consider values $(a_1, a_2, \ldots, a_n)$ as arguments of aggregation with an associated weight vector $\boldsymbol{w}$.

Definition 2.1 (*Compensative Weighted Averaging*). A compensative weighted averaging (CWA) operator (Aggarwal, 2015) of dimension n is a mapping $\mathrm{CWA}: \mathbb{R}^n \to \mathbb{R}$ defined by arguments $(a_1, a_2, \ldots, a_n)$, an associated weight vector $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ such that $\sum_{i=1}^n w_i = 1$ and $w_i \in [0,1]$, and a parameter $0 < \lambda \leqslant \infty, \lambda \neq 1$. The aggregated value is obtained as

$$\mathrm{CWA}(a_1, a_2, \ldots, a_n) = \log_\lambda \left(\sum_{i=1}^n w_i \lambda^{a_i} \right) \tag{1}$$

In order to emphasize the parameters $\boldsymbol{w}$ and λ, at times we shall write CWA as $\mathrm{CWA}_{\boldsymbol{w},\lambda}$. We take the following examples to show the main characteristics of CWA operator.

Example 2.1. Apply CWA operator to aggregate values $\boldsymbol{a} = (8, 4, 6, 10)$, with the weight vector $\boldsymbol{w} = (0.4, 0.3, 0.2, 0.1)$, and $\lambda = 10$. Compare the results obtained with those obtained through weighted average.

The aggregated value is obtained as

$$\mathrm{CWA}_{10}(8,4,6,10) = \log_{10}((0.4*(10)^8) + (0.3*(10)^4) + (0.2*(10)^6) + (0.1*(10)^{10})) = 9.01$$

It can be observed that the aggregated value lies between the minimum (4) and the maximum (10) values. In comparison, the weighted average for the given values is $\mathrm{WA}(8,4,6,10) = 6.60$.

Example 2.2. We compare the results for the Example 2.1 by taking $\lambda = 10^{-29}, \lambda = 1.00001$ and $\lambda = 10^{29}$.

$\mathrm{CWA}_{10^{-29}}(8,4,6,10) = 4.01$; $\mathrm{CWA}_{1.00001}(8,4,6,10) = 6.60$; $\mathrm{CWA}_{10^{29}}(8,4,6,10) = 9.96$

It can be clearly seen that at $\lambda = 1.00001$, we have the aggregated value as 6.60, equal to the weighted mean. And, at $\lambda = 10^{-29}$ and $\lambda = 10^{29}$, we get the aggregation results as 4.01 and 9.96, respectively, which are close to the minimum and maximum of the values to be aggregated.

CWA operators are commutative, monotonic, bounded and idempotent. They have limit cases as $\min_i\{a_i\}$ and $\max_i\{a_i\}$ that are obtained at $\lambda \in (0, \infty]$ taking the extreme values of the interval. They provide a range of operators depending upon the values of λ and $\boldsymbol{w}$, including the simple weighted average as a special case. As the value of parameter $\lambda \in (0, \infty]$ moves towards the extremes of the interval $(0, \infty]$, the aggregation result moves towards the non-compensatory "minimum" or the fully compensatory "maximum" in these operators.

3. Generalized compensative weighted averaging operators

We generalize CWA operator with generalized means (Dyckhoff & Pedrycz, 1984) to develop a new class of powerful compensatory aggregation operators that we term as generalized compensative weighted averaging (GCWA) operators. These operators along with their compensatory capabilities are investigated in this section.

Definition 3.1 (*GCWA*). A generalized compensative weighted averaging (GCWA) operator of dimension n is a mapping $\mathrm{GCWA}: \mathbb{R}_+^n \to \mathbb{R}_+$, given as

$$\mathrm{GCWA}(a_1, a_2, \ldots, a_n) = \left(\log_\lambda \left(\sum_{i=1}^n w_i \lambda^{(a_i)^p} \right) \right)^{\frac{1}{p}} \tag{2}$$

where p is a parameter such that $p \in [-\infty, \infty], p \neq 0$; $0 < \lambda \leqslant \infty$; n is the count of the values to be aggregated; and $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ is a collection of weights such that $\sum_{i=1}^n w_i = 1$ and $w_i \in [0,1]$.

Note 1. At times, we shall indicate GCWA operator as $\mathrm{GCWA}_{p,\lambda,\boldsymbol{w}}$ to emphasize the parameters.

From (2), it can be observed that GCWA generalizes CWA operator with an additional parameter that controls the power to which the argument values are raised. An infinite number of possible combinations of parameters λ and p in GCWA operator lead to a very fine range of compensatory effects. Since GCWA operator is a generalization of CWA aggregation operator, it has the characteristics of both CWA operator and the generalized means.

The GCWA operator has $\min_i\{a_i\}$ and $\max_i\{a_i\}$, as it's limit cases. There are a large number of special cases of GCWA operator, which we explore in depth. Two of them are of great significance. First is the case, at $p = 1$, when GCWA operator reduces to CWA operator:

$$\mathrm{GCWA}_{1,\lambda,\boldsymbol{w}}(a_1, a_2, \ldots, a_n) = \log_\lambda \left(\sum_{i=1}^n w_i \lambda^{(a_i)} \right) = \mathrm{CWA}_{\lambda,\boldsymbol{w}}(a_1, a_2, \ldots, a_n)$$

The other important special cases arise at $\lambda \to 1$, leading to the weighted exponential means viz. arithmetic, harmonic, quadratic, and other higher order means, corresponding to $p = 1, p = -1, p = 2, \ldots$, respectively. We investigate these special cases in Section 4. We now show that GCWA operator exhibits the properties of commutativity, boundness, idempotency and monotonicity.

Theorem 3.1 (*Commutativity*). *Let $(d_1, d_2, \ldots, d_n)$ be any permutation of arguments $(a_1, a_2, \ldots, a_n)$. Then*

$$\mathrm{GCWA}(a_1, a_2, \ldots, a_n) = \mathrm{GCWA}(d_1, d_2, \ldots, d_n) \tag{3}$$

Proof. From the definition of GCWA operator in (2), we get

$$\mathrm{GCWA}(a_1, a_2, \ldots, a_n) = \left(\log_\lambda \left(\sum_{i=1}^n w_i \lambda^{(a_i)^p} \right) \right)^{\frac{1}{p}}$$

$$\mathrm{GCWA}(d_1, d_2, \ldots, d_n) = \left(\log_\lambda \left(\sum_{i=1}^n w_i \lambda^{(d_i)^p} \right) \right)^{\frac{1}{p}}$$

Since, the value of p and the association of the weight vector with the arguments remains intact in the permutation $(d_1, d_2, \ldots, d_n)$, as in the $(a_1, a_2, \ldots, a_n)$. Therefore,

$$\mathrm{GCWA}(a_1, a_2, \ldots, a_n) = \log_\lambda \left(\sum_{i=1}^n w_i \lambda^{a_i} \right) = \log_\lambda \left(\sum_{i=1}^n w_i \lambda^{d_i} \right) = \mathrm{GCWA}(d_1, d_2, \ldots, d_n) \quad \square$$

Theorem 3.2 (*Boundness*). *Let $\max_i\{a_i\} = M$ and $\min_i\{a_i\} = m$. Then*

$$\min_i\{a_i\} \leqslant \mathrm{GCWA}(a_1, a_2, \ldots, a_n) \leqslant \max_i\{a_i\} \tag{4}$$

Proof. Since, $\sum_{i=1}^n w_i = 1$,